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PAPER

Filter Design for IBI Suppression in OFDM Based Filter-and-Forward Relay Beamforming

Satoshi NAGAI[†*], ***Student Member and*** **Teruyuki MIYAJIMA**[††a)], ***Member***

SUMMARY In this paper, we consider filter-and-forward relay beamforming using orthogonal frequency-division multiplexing (OFDM) in the presence of inter-block interference (IBI). We propose a filter design method based on a constrained max-min problem, which aims to suppress IBI and also avoid deep nulls in the frequency domain. It is shown that IBI can be suppressed completely owing to the employment of beamforming with multiple relays or multiple receive antennas at each relay when perfect channel state information (CSI) is available. In addition, we modify the proposed method to cover the case where only the partial CSI for relay-receiver channels is available. Numerical simulation results show that the proposed method significantly improves the performance as the number of relays and antennas increases due to spatial diversity, and the modified method can make use of the channel correlation to improve the performance.

key words: *OFDM, filter-and-forward, inter-block interference, time-domain equalizer, second order statistics*

1. Introduction

In wireless networks, relay technologies are effective to enlarge the coverage area and to enhance the system performance by sophisticated signal processing in relays and cooperative diversity [1]. Relay techniques have been adopted in wireless standards such as 3GPP LTE-Advanced [2]. Among the various relaying schemes [3], [4], the amplify-and-forward (AF) scheme [5] is the most popular scheme; the relay amplifies the signal received from the transmitter and transmits the amplified signal to the receiver. In frequency-selective channels, the AF scheme is combined with orthogonal frequency-division multiplexing (OFDM) where inter-symbol interference (ISI) can be eliminated by inserting a cyclic prefix (CP) between OFDM blocks.

An extension of AF is known as the filter-and-forward (FF) scheme [6], [7], where the received signal at a relay is passed through a linear filter, and then transmitted to the receiver. Most previous studies on the FF scheme have focused on ISI suppression in single-carrier systems where the linear filter works as a time-domain equalizer (TEQ) for the ISI suppression. Recently, the authors in [8] have considered the application of the FF scheme to OFDM systems. Unlike the AF scheme, the FF scheme dispenses with OFDM demodulation and remodulation at a relay. Furthermore, unlike single-carrier systems, the filter at a relay can be devoted to enhancing the system performance in the OFDM systems where ISI is no longer a problem.

An overlooked problem in the previous works on the FF scheme is that the use of TEQ at relays could introduce undesired interference. The total channel from the transmitter to the receiver consists of three linear systems, i.e., transmitter-relay channel, TEQ at a relay, and relay-receiver channel, which lengthens the channel's impulse response. If the impulse response length of the total channel exceeds the CP length, undesired inter-block interference (IBI) occurs, and system performance is seriously degraded.

To overcome this problem, the authors in [9], [10] proposed the shorten-and-forward scheme where TEQ at relays performs channel shortening [11], [12], which shortens the total channel impulse response to within the CP length. In [9], [10], a single relay with a single receive antenna was deployed, and a classic filter design, namely the maximization of shortened-SNR (MSSNR) [11], was employed. For this approach, there are three issues to be addressed: first, a single relay with a single receive antenna cannot suppress IBI perfectly as shown in this paper; second, the performance of MSSNR, which is a time-domain design, is limited since it is susceptible to deep nulls in the frequency domain; third, although this method assumes that the perfect channel state information (CSI) of relay-receiver channels is available, it is not always true in practice.

In this paper, we propose new filter design methods for OFDM-based FF relay networks. Unlike [9], [10], we apply the proposed method to the case where multiple FF relays with multiple receive antennas are deployed. It is shown that IBI can be suppressed by increasing either the number of relays or that of receive antennas at each relay when the perfect CSI is available. Also, instead of maximizing the SSNR in time-domain, the proposed method maximizes the worst subcarrier gain of the total channel in the frequency domain to avoid deep nulls. Simulation results show that this approach successfully improves the performance compared to the MSSNR approach. Moreover, we modify the proposed method to make it applicable to the case where only the partial CSI of relay-receiver channels, i.e., second-order statistics (SOS), can be accessed. The effectiveness of the modified method is verified through simulation results.

The rest of this paper is organized as follows. Section 2 describes an OFDM-based FF network model. In Sect. 3, we propose a filter design method when the perfect CSI is avail-

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[†]The author is with the Graduate School of Science and Engineering, Ibaraki University, Hitachi-shi, 316-8511 Japan.
[††]The author is with the Department of Electrical and Electronic Engineering, Ibaraki University, Hitachi-shi, 316-8511 Japan.
[*]Presently, with Hitachi Ltd.
a) E-mail: teruyuki.miyajima.spc@vc.ibaraki.ac.jp

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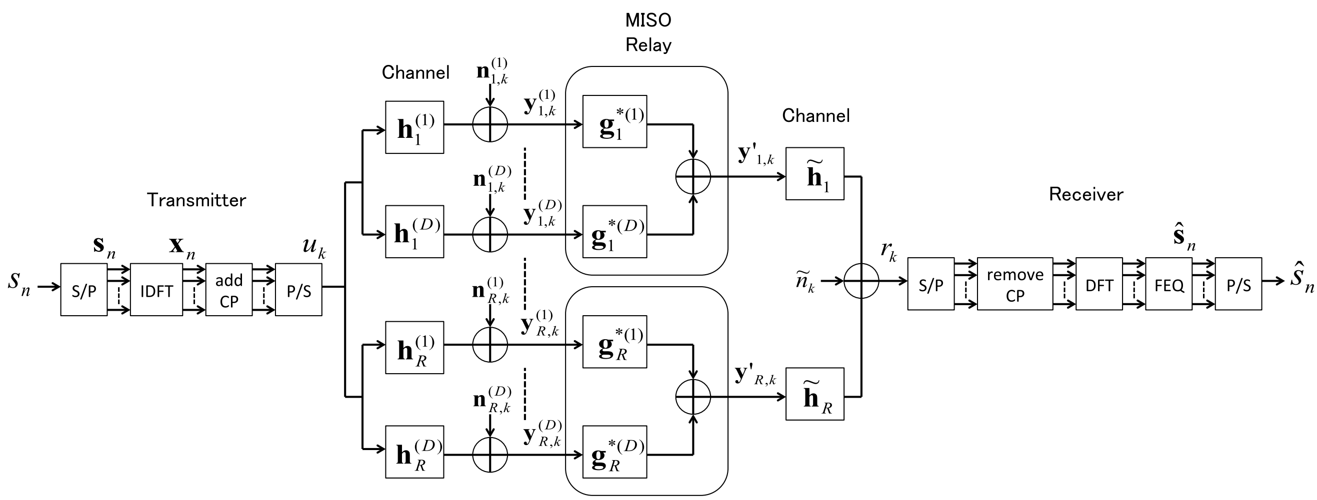

Fig. 1 OFDM-based FF relay beamforming model.

able, and its modification to the case with the partial CSI is presented in Sect. 4. Section 5 shows simulation results, followed by conclusions in Sect. 6.

Throughout this paper, we use the following notations: $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^*$ represent the transpose, conjugate transpose, conjugate of a vector (matrix), respectively. In addition, $\mathrm{diag}(\mathbf{A}_1 \ \cdots \ \mathbf{A}_N)$ indicates a block-diagonal matrix with the matrices $\mathbf{A}_1, \ \cdots, \ \mathbf{A}_N$ on the main diagonal, trace($\mathbf{A}$) indicates the trace of $\mathbf{A}$, vec($\mathbf{A}$) forms a vector by stacking columns of $\mathbf{A}$, $\otimes$ indicates the Kronecher product, $\mathbf{I}_N$ denotes an identity matrix of size N, and $\mathbf{0}_{a\times b}$ indicates an $a \times b$ zero matrix.

2. System Model

2.1 Transmitter

We consider an OFDM-based FF relay network where there are R relays and each relay has D receive antennas as shown in Fig. 1. In the transmitter, data symbols are converted by S/P-conversion, and the nth data symbol block consisting of N symbols is represented as

$$\mathbf{s}_n = \begin{bmatrix} s_{nN} & \cdots & s_{(n+1)N-1} \end{bmatrix}^T \tag{1}$$

where s_k is an i.i.d. complex-valued data sequence with variance σ_s^2. Applying the inverse discrete Fourier transform (IDFT) to $\mathbf{s}_n$ produces a time-domain block vector of length N

$$\mathbf{x}_n = \begin{bmatrix} x_{n,0} & \cdots & x_{n,N-1} \end{bmatrix}^T = \mathbf{F}^H \mathbf{s}_n \tag{2}$$

where $\mathbf{F}$ is an N-dimensional DFT matrix that has a unitary property $\mathbf{F}\mathbf{F}^H = \mathbf{I}_N$:

$$\mathbf{F} = \frac{1}{\sqrt{N}} \begin{bmatrix} e^{j0} & e^{j0} & \cdots & e^{j0} \\ e^{j0} & e^{-\frac{j2\pi}{N}} & \cdots & e^{-\frac{j2(N-1)\pi}{N}} \\ \vdots & \vdots & \ddots & \vdots \\ e^{j0} & e^{-\frac{j2\pi}{N}} & \cdots & e^{-\frac{j2(N-1)(N-1)\pi}{N}} \end{bmatrix}. \tag{3}$$

A CP of length P is added to the top of $\mathbf{x}_n$ to form the nth transmitted block $\mathbf{u}_n$ whose ith entry is given by

$$u_{nQ+i} = x_{n,(i+N-P)_N} \tag{4}$$

where $i = 0, 1, \ \cdots, \ Q-1, Q = N+P$, and $(m)_N$ represents the reminder of m modulo N. After P/S-conversion, the sequence u_k is transmitted.

2.2 Filter-and-Forward Relays

The transmitted signal passes through frequency selective channels from the transmitter to relays and is received by R relays with D receive antennas. The received signal at the dth antenna of the rth relay at time k is given by

$$y_{r,k}^{(d)} = \sum_{m=0}^{M} h_{r,m}^{(d)} u_{k-m} + n_{r,k}^{(d)} \tag{5}$$

where $h_{r,m}^{(d)}$ is the impulse response of the channel from the transmitter to the dth antenna of the rth relay, M is the order of channels, and $n_{r,k}^{(d)}$ is a complex-valued additive white Gaussian noise. The received signal $y_{r,k}^{(d)}$ is fed into the dth TEQ of length L, and the total MISO-TEQ output is given by

$$y'_{r,k} = \sum_{d=1}^{D} \sum_{l=0}^{L-1} g_{r,l}^{(d)} y_{r,k-l}^{(d)} \tag{6}$$

where $g_{r,l}^{(d)}$ is the impulse response of the dth TEQ of the rth relay. The total TEQ output is transmitted from a single transmit antenna of the rth relay.

For later convenience, the above quantities are represented by vector form. The received signal vector at the dth antenna of the rth relay can be written as

$$\begin{aligned} \mathbf{y}_{r,k}^{(d)} &= \begin{bmatrix} y_{r,k}^{(d)} & \cdots & y_{r,k-(L+M)+1}^{(d)} \end{bmatrix}^T \\ &= \mathbf{H}_r^{(d)} \mathbf{u}_k + \mathbf{n}_{r,k}^{(d)} \end{aligned} \tag{7}$$

where $\mathbf{u}_k = [u_k \ \cdots \ u_{k-(L+2M)+1}]^T$ is a transmitted signal vector, $\mathbf{n}_{r,k}^{(d)} = [n_{r,k}^{(d)} \ \cdots \ n_{r,k-(L+M)+1}^{(d)}]^T$ is a noise vector, and $\mathbf{H}_r^{(d)} \in \mathbb{C}^{(L+M)\times(L+2M)}$ is a channel matrix given by

$$\mathbf{H}_r^{(d)} = \text{Toeplitz}(\mathbf{h}_r^{(d)T}, L+M) = \begin{bmatrix} h_{r,0}^{(d)} & \cdots & h_{r,M}^{(d)} & 0 & \cdots & 0 \\ 0 & h_{r,0}^{(d)} & \cdots & h_{r,M}^{(d)} & 0 & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & h_{r,0}^{(d)} & \cdots & h_{r,M}^{(d)} \end{bmatrix} \quad (8)$$

where the notation Toeplitz$(\mathbf{a}, m)$ denotes a Toeplitz matrix with m rows and $[\mathbf{a} \ \mathbf{0}_{1\times(m-1)}]$ as the first row, and $\mathbf{h}_r^{(d)} = [h_{r,0}^{(d)} \ \cdots \ h_{r,M}^{(d)}]^T$. The rth TEQ output vector can be written as

$$\begin{aligned} \mathbf{y}'_{r,k} &= \left[y'_{r,k} \ \cdots \ y'_{r,k-M}\right]^T \\ &= \sum_{d=1}^{D} \mathbf{G}_r^{(d)*}\mathbf{y}_{r,k}^{(d)} \\ &= \sum_{d=1}^{D} \mathbf{G}_r^{(d)*}\left(\mathbf{H}_r^{(d)}\mathbf{u}_k + \mathbf{n}_{r,k}^{(d)}\right) \\ &= \mathbf{G}_r^*\mathbf{H}_r\mathbf{u}_k + \mathbf{G}_r^*\mathbf{n}_{r,k} \end{aligned} \quad (9)$$

where $\mathbf{G}_r^{(d)} = \text{Toeplitz}(\mathbf{g}_r^{(d)T}, M+1) \in \mathbb{C}^{(M+1)\times(L+M)}$ is a TEQ matrix, $\mathbf{g}_r^{(d)} = [g_{r,0}^{(d)} \ \cdots \ g_{r,L-1}^{(d)}]^T$ is the impulse response vector of the dth TEQ of the rth relay, $\mathbf{G}_r = [\mathbf{G}_r^{(1)} \ \cdots \ \mathbf{G}_r^{(D)}] \in \mathbb{C}^{(M+1)\times(L+M)D}$, $\mathbf{H}_r = [\mathbf{H}_r^{(1)T} \ \cdots \ \mathbf{H}_r^{(D)T}]^T \in \mathbb{C}^{(L+M)D\times(L+2M)}$, and $\mathbf{n}_{r,k} = [\mathbf{n}_{r,k}^{(1)T} \ \cdots \ \mathbf{n}_{r,k}^{(D)T}]^T$.

2.3 Receiver

The transmitted signal from each relay passes through a frequency-selective channel and are received by a single receive antenna equipped at the receiver. The received signal at time k is given by

$$r_k = \sum_{r=1}^{R}\sum_{m=0}^{M} \tilde{h}_{r,m} y'_{r,k-m} + \tilde{n}_k \quad (10)$$

where $\tilde{h}_{r,m}$ is the impulse response of the channel between the rth relay and the receiver, and $\tilde{n}_k$ represents an additive noise at the receiver. The received signal can be rewritten using a vector notation as

$$\begin{aligned} r_k &= \sum_{r=1}^{R} \tilde{\mathbf{h}}_r^T \mathbf{y}'_{r,k} + \tilde{n}_k \\ &= \sum_{r=1}^{R} \tilde{\mathbf{h}}_r^T \left(\mathbf{G}_r^*\mathbf{H}_r\mathbf{u}_k + \mathbf{G}_r^*\mathbf{n}_{r,k}\right) + \tilde{n}_k \\ &= \tilde{\mathbf{h}}^T\mathbf{G}^*\mathbf{H}\mathbf{u}_k + \tilde{\mathbf{h}}^T\mathbf{G}^*\mathbf{n}_k + \tilde{n}_k \end{aligned} \quad (11)$$

where $\tilde{\mathbf{h}} = [\tilde{\mathbf{h}}_1^T \ \cdots \ \tilde{\mathbf{h}}_R^T]^T$, $\tilde{\mathbf{h}}_r = [\tilde{h}_{r,0} \ \cdots \ \tilde{h}_{r,M}]^T$, $\mathbf{G} = \text{diag}(\mathbf{G}_1 \ \cdots \ \mathbf{G}_R) \in \mathbb{C}^{(M+1)R\times(L+M)DR}$, $\mathbf{H} = [\mathbf{H}_1^T \ \cdots \ \mathbf{H}_R^T]^T \in \mathbb{C}^{(L+M)DR\times(L+2M)}$, and $\mathbf{n}_k = [\mathbf{n}_{1,k}^T \ \cdots \ \mathbf{n}_{R,k}^T]^T$. After removing the CP from the received signal r_k, the N-point DFT and a frequency-domain equalizer (FEQ) are applied, and finally we get demodulated symbols $\hat{\mathbf{s}}_n$.

3. IBI Suppression Filter Design Using Full CSI

3.1 Formulation of the Optimization Problem

We assume that we have the perfect knowledge on all channels $\{h_{r,k}^{(d)}, \tilde{h}_{r,k}\}$. Our goal is to find $\mathbf{g} = [\mathbf{g}_1^T \ \cdots \ \mathbf{g}_R^T]^T$, which is the TEQ vector of length $L_0 = LDR$ where $\mathbf{g}_r = [\mathbf{g}_r^{(1)T} \ \cdots \ \mathbf{g}_r^{(D)T}]^T$, such that IBI is canceled completely and deep nulls in the frequency domain are avoided.

Now, we propose a method to determine a TEQ vector $\mathbf{g}$:

$$\begin{aligned} &\max_{\mathbf{g}} \ \min_{i} \ P_i^{\text{desired}} \\ &\text{s.t.} \ \ P_i^{\text{IBI}} = 0, \forall i \end{aligned} \quad (12)$$

where P_i^{desired} is the power of the desired signal component at the ith subcarrier, and P_i^{IBI} is the power of the IBI component at the ith subcarrier. In (12), the minimum power of the desired signal component over subcarriers is maximized to avoid deep nulls in the frequency domain. Furthermore, we impose the constraints to suppress the IBI component. It is shown later that the constraints ensure the complete IBI suppression if $L_0 > L + 2M - P - 1$.

In the following, we derive P_i^{desired} and P_i^{IBI}. Let us define a composite channel $c_{r,k}^{(d)}$ consisting of two physical channels $h_{r,k}^{(d)}$ and $\tilde{h}_{r,k}$ as

$$c_{r,k}^{(d)} = \sum_{m=0}^{M} h_{r,m}^{(d)}\tilde{h}_{r,k-m} \quad (13)$$

for $k = 0, 1, \cdots, 2M$, and form a vector $\mathbf{c}_r^{(d)} = [c_{r,0}^{(d)} \ \cdots \ c_{r,2M}^{(d)}]^T$. Since we assume that there exists IBI, it holds that $2M > P$. Furthermore, the total channel from the transmitter to the receiver can be written in a vector as

$$\begin{aligned} \mathbf{t} &= [t_0 \ t_1 \ \cdots \ t_{L+2M-1}] \\ &= \sum_{r=1}^{R}\sum_{d=1}^{D} \mathbf{g}_r^{(d)H}\mathbf{C}_r^{(d)} \\ &= \sum_{\theta=1}^{R} \mathbf{g}_r^H\mathbf{C}_r \\ &= \mathbf{g}^H\mathbf{C} \end{aligned} \quad (14)$$

where $\mathbf{C}_r^{(d)} = \text{Toeplitz}(\mathbf{c}_r^{(d)T}, L) \in \mathbb{C}^{L\times(L+2M)}$, $\mathbf{C}_r = [\mathbf{C}_r^{(1)T} \ \cdots \ \mathbf{C}_r^{(D)T}]^T \in \mathbb{C}^{LD\times(L+2M)}$, and $\mathbf{C} = [\mathbf{C}_1^T \ \cdots \ \mathbf{C}_R^T]^T \in \mathbb{C}^{L_0\times(L+2M)}$. Then, the received signal in (11) can be rewritten as

$$r_k = \mathbf{t}\mathbf{u}_k + \tilde{\mathbf{h}}^T\mathbf{G}^*\mathbf{n}_k + \tilde{n}_k. \quad (15)$$

In the following, we assume that $L+2M \leq N$, which ensures that IBI comes from only an adjacent block, and that the transmitted signal can be regarded as stationary.

The impulse response vector of the total channel can be divided into the desired and the remaining IBI component as

$$\mathbf{t}^{\text{desired}} = \mathbf{t}\tilde{\mathbf{J}} = \mathbf{g}^H\mathbf{C}\tilde{\mathbf{J}} \quad (16)$$
$$\mathbf{t}^{\text{IBI}} = \mathbf{t}\tilde{\mathbf{T}} = \mathbf{g}^H\mathbf{C}\tilde{\mathbf{T}} \quad (17)$$

where $(L+2M)\times(L+2M)$ diagonal matrices $\tilde{\mathbf{J}}$ and $\tilde{\mathbf{T}}$ are defined by

$$\tilde{\mathbf{J}} = \text{diag}(\underbrace{0\ \cdots\ 0}_{\delta}\ \underbrace{1\ \cdots 1}_{P+1}\ 0\ \cdots\ 0) \quad (18)$$
$$\tilde{\mathbf{T}} = \text{diag}(\underbrace{1\ \cdots\ 1}_{\delta}\ \underbrace{0\ \cdots\ 0}_{P+1}\ 1\ \cdots\ 1) \quad (19)$$

where δ is a decision delay which is chosen from the range $0 \le \delta \le L+2M-P-1$. The delay controls which part of the channel contributes to the desired component in the total channel, and thus the resulting desired component with a proper choice of δ can be large. The frequency response of the desired component and the IBI component at the ith subcarrier are given by [13]

$$T_i^{\text{desired}} = \mathbf{g}^H\mathbf{C}\mathbf{J}\mathbf{q}_i \quad (20)$$
$$T_i^{\text{IBI}} = \mathbf{g}^H\mathbf{C}\mathbf{T}\mathbf{q}_i \quad (21)$$

where $(L+2M)\times N$ matrices $\mathbf{J}$ and $\mathbf{T}$ are given by $\mathbf{J} = \begin{bmatrix}\tilde{\mathbf{J}}\ \mathbf{0}\end{bmatrix}$ and $\mathbf{T} = \begin{bmatrix}\tilde{\mathbf{T}}\ \mathbf{0}\end{bmatrix}$, and $\mathbf{q}_i^T = \frac{1}{\sqrt{N}}[e^{j0}\ \ e^{\frac{-j2\pi i}{N}}\ \cdots\ e^{\frac{-j2\pi(N-1)i}{N}}]$ is the ith row of the DFT matrix $\mathbf{F}$. From (20), P_i^{desired} is given by

$$P_i^{\text{desired}} = S_{u,i} \mid T_i^{\text{desired}} \mid^2 = \mathbf{g}^H\mathbf{A}_i\mathbf{g} \quad (22)$$

where $S_{u,i} = \sigma_s^2$ is the power of the transmitted signal u_k at the ith subcarrier and

$$\mathbf{A}_i = \sigma_s^2\mathbf{C}\mathbf{J}\mathbf{q}_i\mathbf{q}_i^H\mathbf{J}^H\mathbf{C}^H. \quad (23)$$

Similarly, P_i^{IBI} is given by

$$P_i^{\text{IBI}} = S_{u,i} \mid T_i^{\text{IBI}} \mid^2 = \mathbf{g}^H\mathbf{B}_i\mathbf{g} \quad (24)$$

where

$$\mathbf{B}_i = \sigma_s^2\mathbf{C}\mathbf{T}\mathbf{q}_i\mathbf{q}_i^H\mathbf{T}^H\mathbf{C}^H. \quad (25)$$

Substituting (22) and (24) into (12), we have the following optimization problem:

$$\begin{aligned} \max_{\mathbf{g}}\ \ & \min_i\ \mathbf{g}^H\mathbf{A}_i\mathbf{g} \\ \text{s.t.}\ \ & \mathbf{g}^H\mathbf{B}_i\mathbf{g} = 0, \forall i. \end{aligned} \quad (26)$$

The TEQ vector $\mathbf{g}$ is obtained by solving this problem.

3.2 Proposed Method

Let us briefly explain how the constrained optimization problem (26) is solved. To solve the problem efficiently, we take the approach in [14]. Consider a positive semidefinite matrix $\tilde{\mathbf{G}} = \mathbf{g}\mathbf{g}^H$ of rank 1. Then, the problem in (26) can be expressed as

$$\begin{aligned} \max_{\tilde{\mathbf{G}}}\ \ & \min_i\ \text{trace}(\tilde{\mathbf{G}}\mathbf{A}_i) \\ \text{s.t.}\ \ & \text{trace}(\tilde{\mathbf{G}}\mathbf{B}_i) = 0, \forall i \\ & \text{rank}(\tilde{\mathbf{G}}) = 1 \\ & \tilde{\mathbf{G}} \ge 0 \end{aligned} \quad (27)$$

where $\tilde{\mathbf{G}} \ge 0$ implies that $\tilde{\mathbf{G}}$ is positive semidefinite. An approach to solving the problem with a non-convex rank-one constraint approximately is to apply semidefinite relaxation by dropping the rank-one constraint. Using nonnegative variables τ, the above problem can be reformulated as a convex semidefinite programming (SDP):

$$\begin{aligned} \max\ \ & \tau \\ \text{s.t.}\ \ & \text{trace}(\bar{\mathbf{G}}\mathbf{A}_i) \ge \tau \\ & \text{trace}(\bar{\mathbf{G}}\mathbf{B}_i) = 0, \forall i \\ & \bar{\mathbf{G}} \ge 0. \end{aligned} \quad (28)$$

The proposed method is summarized as follows: first, the matrices $\mathbf{A}_i$, and $\mathbf{B}_i$ are computed by (23) and (25); second, the problem (28) is solved by using an SDP solver such as SeDuMi [15] together with the modeling language YALMIP [16], and a solution $\bar{\mathbf{G}}$ is obtained; third, the randomization procedure [14] is performed to obtain a TEQ vector, i.e., the best solution is chosen from the candidates $\mathbf{g}_p = \mathbf{U}\mathbf{\Sigma}^{1/2}\mathbf{v}_p$, $p = 1, 2, \cdots, N_p$, where $\mathbf{U}$ and $\mathbf{\Sigma}$ are a matrix consisting of eigenvectors and a diagonal matrix with eigenvalues of $\bar{\mathbf{G}} = \mathbf{U}\mathbf{\Sigma}\mathbf{U}^H$, respectively, and $\mathbf{v}_p$ is a vector of complex Gaussian random variables with zero-mean and unit-variance.

3.3 IBI Suppression Capability

We further investigate the constraints in (26). In the following, we omit σ_s^2 without loss of generality. Since $\mathbf{B}_i$ in (25) is a positive semi-definite matrix, the constraints in (26) can be rewritten as

$$\mathbf{g}^H\mathbf{B}_i\mathbf{g} = 0, \forall i \Longleftrightarrow \mathbf{g}^H\left(\sum_{i=0}^{N-1}\mathbf{B}_i\right)\mathbf{g} = 0. \quad (29)$$

Furthermore, we have

$$\begin{aligned} \mathbf{B} = \sum_{i=0}^{N-1}\mathbf{B}_i &= \mathbf{C}\mathbf{T}\left(\sum_{i=0}^{N-1}\mathbf{q}_i\mathbf{q}_i^H\right)\mathbf{T}^H\mathbf{C}^H \\ &= \mathbf{C}\mathbf{T}\mathbf{F}\mathbf{F}^H\mathbf{T}^H\mathbf{C}^H \\ &= \mathbf{C}\mathbf{T}\mathbf{T}^H\mathbf{C}^H \\ &= \mathbf{C}\tilde{\mathbf{T}}\tilde{\mathbf{T}}^H\mathbf{C}^H. \end{aligned} \quad (30)$$

Thus, the constraints imply that $||\mathbf{g}^H\mathbf{C}\tilde{\mathbf{T}}||^2 = ||\mathbf{t}^{\text{IBI}}||^2 = 0$, i.e., IBI can be suppressed completely. Now, we consider if there exists $\mathbf{g}$ such that the constraints in (26) are satisfied. Let $\lambda_0 \ge \lambda_1 \ge \cdots \ge \lambda_{L_0-1}$ be the eigenvalues of $\mathbf{B}$. Since the rank of $\tilde{\mathbf{T}}$ is $L+2M-P-1$, the rank of the matrix product $\mathbf{C}\tilde{\mathbf{T}}$ does not exceed $L+2M-P-1$. As a result, the rank of $\mathbf{B} \in \mathbb{C}^{L_0\times L_0}$ is also less than or equal to $\min(L_0, L+2M-P-1)$. Therefore, if we have

$$L_0 > L + 2M - P - 1, \tag{31}$$

the eigenvalues of $\mathbf{B}$ becomes $\lambda_0 \geq \lambda_1 \geq \cdots \geq \lambda_{L+2M-P-1} = \cdots = \lambda_{L_0-1} = 0$. Then, if we choose $\mathbf{g}$ from the null subspace of $\mathbf{B}$, the constraints in (26) are satisfied. Note that the sufficient condition in (31) can be easily satisfied by increasing the number of antennas D or relays R.

In the case of a single relay with a single antenna [10], the condition in (31) cannot be satisfied. When $D = 1$ and $R = 1$, $\mathbf{C}\tilde{\mathbf{T}}$ has full-row rank of L. Then $\mathbf{B}$ has full rank, and thus $\mathbf{g}^H\mathbf{B}\mathbf{g} \neq 0$ for any $\mathbf{g}$, i.e., residual IBI remains.

4. IBI Suppression Filter Design Using SOS

In the previous section, we assumed that the perfect CSI, i.e., instantaneous channel impulse response is available. However, it is not easy to obtain the perfect CSI, especially for the relay-receiver channels $\tilde{\mathbf{h}}$. In this section, we modify the proposed method to make it applicable to the case where only the second-order statistics (SOS) of the channels, $E[\tilde{\mathbf{h}}\tilde{\mathbf{h}}^H]$, is available.

4.1 Alternative Expression for the Desired and IBI Components

In (22) and (24), the composite channel $c_{r,k}^{(d)}$, which consists of $h_{r,k}^{(d)}$ and $\tilde{h}_{r,k}$, is used to express P_i^{desired} and P_i^{IBI}. However, the composite channel cannot be used unless the instantaneous channel impulse response $\tilde{h}_{r,k}$ is known. Here, we provide the average desired component power and the average IBI component power without $c_{r,k}^{(d)}$.

The desired component at the ith subcarrier in (20) can also be expressed as $T_i^{\text{desired}} = \tilde{\mathbf{h}}^T\mathbf{G}^*\mathbf{H}\mathbf{J}\mathbf{q}_i$ from (11). Then, the average power of the ith desired component becomes †

$$\begin{aligned}
E\left[P_i^{\text{desired}}\right] &= S_{u,i}E\left[|\ T_i^{\text{desired}}\ |^2\right] \\
&= \sigma_s^2\mathbf{q}_i^H\mathbf{J}^H\mathbf{H}^H\mathbf{G}^T \underbrace{E\left[\tilde{\mathbf{h}}^*\tilde{\mathbf{h}}^T\right]}_{\mathbf{\Phi}^*}\mathbf{G}^*\mathbf{H}\mathbf{J}\mathbf{q}_i \\
&= \sigma_s^2\text{trace}(\mathbf{q}_i^H\mathbf{J}^H\mathbf{H}^H\mathbf{G}^T\mathbf{\Phi}^*\mathbf{G}^*\mathbf{H}\mathbf{J}\mathbf{q}_i) \\
&= \sigma_s^2\text{trace}(\mathbf{G}^*\tilde{\mathbf{A}}_i\mathbf{G}^T\mathbf{\Phi}^*) \\
&= \sigma_s^2[\text{vec}(\mathbf{G}^H)]^T\bar{\mathbf{A}}_i\text{vec}(\mathbf{G}^T) \\
&= \sigma_s^2\mathbf{g}^H\mathbf{E}\bar{\mathbf{A}}_i\mathbf{E}^H\mathbf{g} \\
&= \mathbf{g}^H\mathbf{A}_i\mathbf{g}
\end{aligned} \tag{32}$$

where

$$\tilde{\mathbf{A}}_i = \mathbf{H}\mathbf{J}\mathbf{q}_i\mathbf{q}_i^H\mathbf{J}^H\mathbf{H}^H \in \mathbb{C}^{l_DR\times l_DR}, \tag{33}$$

$$\bar{\mathbf{A}}_i = \mathbf{\Phi}^H \otimes \tilde{\mathbf{A}}_i \in \mathbb{C}^{l_D(M+1)R^2\times l_D(M+1)R^2}, \tag{34}$$

$$\mathbf{A}_i = \sigma_s^2\mathbf{E}\bar{\mathbf{A}}_i\mathbf{E}^H \in \mathbb{C}^{L_0\times L_0}. \tag{35}$$

Moreover, $\mathbf{E}$ is given by

$$\mathbf{E} = \text{diag}\left(\tilde{\mathbf{V}}\tilde{\mathbf{E}}_1\ \cdots\ \tilde{\mathbf{V}}\tilde{\mathbf{E}}_R\right) \in \mathbb{R}^{L_0\times l_D(M+1)R^2} \tag{36}$$

†trace(**ABCD**) = $[\text{vec}(\mathbf{A}^T)]^T(\mathbf{D}^T\otimes\mathbf{B})\text{vec}(\mathbf{C})$ is used to derive the fourth line from the third line.

where

$$\tilde{\mathbf{V}} = [\mathbf{V}_0\ \cdots\ \mathbf{V}_M] \in \mathbb{R}^{LD\times l_D(M+1)} \tag{37}$$

$$\mathbf{V}_m = [\mathbf{0}_{LD\times m}\ \mathbf{V}\ \mathbf{0}_{LD\times(M-m)}] \in \mathbb{R}^{LD\times l_D} \tag{38}$$

$$\mathbf{V} = \left[\begin{array}{cc|c|cc|c}
\mathbf{I}_L & \mathbf{0}_{L\times M} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \ddots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \ddots & \mathbf{I}_L & \mathbf{0}_{L\times M} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{I}_L
\end{array}\right] \in \mathbb{R}^{LD\times(l_D-M)}$$

$$\tilde{\mathbf{E}}_r = \text{diag}\,(\mathbf{E}_r\ \cdots\ \mathbf{E}_r) \in \mathbb{R}^{l_D(M+1)R} \tag{39}$$

$$\mathbf{E}_r = [\mathbf{0}_{l_D\times l_D(r-1)}\ \mathbf{I}_{l_D}\ \mathbf{0}_{l_D\times l_D(R-r)}] \in \mathbb{R}^{l_D\times l_DR}, \tag{40}$$

and $l_D = (L + M)D$.

Similarly, the frequency response of the IBI component at the ith subcarrier can be expressed as $T_i^{\text{IBI}} = \tilde{\mathbf{h}}^T\mathbf{G}^*\mathbf{H}\mathbf{T}\mathbf{q}_i$. The average power of the ith IBI component is given by

$$E\left[P_i^{\text{IBI}}\right] = S_{u,i}E\left[|\ T_i^{\text{IBI}}\ |^2\right] = \mathbf{g}^H\mathbf{B}_i\mathbf{g} \tag{41}$$

where

$$\tilde{\mathbf{B}}_i = \mathbf{H}\mathbf{T}\mathbf{q}_i\mathbf{q}_i^H\mathbf{T}^H\mathbf{H}^H \in \mathbb{C}^{l_DR\times l_DR}, \tag{42}$$

$$\bar{\mathbf{B}}_i = \mathbf{\Phi}^H \otimes \tilde{\mathbf{B}}_i \in \mathbb{C}^{l_D(M+1)R^2\times l_D(M+1)R^2}, \tag{43}$$

$$\mathbf{B}_i = \sigma_s^2\mathbf{E}\bar{\mathbf{B}}_i\mathbf{E}^H \in \mathbb{C}^{L_0\times L_0}. \tag{44}$$

4.2 Modified Method

In 3.3, we derived a sufficient condition (31) for the perfect IBI suppression in the case of the perfect CSI. As the correlation between elements in $\tilde{\mathbf{h}}$ decreases, the rank of $\mathbf{\Phi}$ increases. For example, $\mathbf{\Phi}$ have full rank if the elements in $\tilde{\mathbf{h}}$ are uncorrelated with each other. Then, $\mathbf{B} = \sum_{i=0}^{N-1}\mathbf{B}_i$ can have full rank even if the condition (31) is satisfied unlike the perfect CSI case, where $\mathbf{B}_i$ is defined in (45) instead of (25). In such a case, $\mathbf{g}^H\mathbf{B}_i\mathbf{g} \neq 0$ for any $\mathbf{g}$, and then the proposed method (26) cannot be used. Thus, we modify the proposed method by relaxing the equality constraints into inequality constraints to allow nonzero IBI as follows:

$$\begin{aligned}
&\max_{\mathbf{g}}\ \min_{i}\ \mathbf{g}^H\mathbf{A}_i\mathbf{g} \\
&\text{s.t.}\quad \mathbf{g}^H\mathbf{B}_i\mathbf{g} \leq \epsilon, \forall i
\end{aligned} \tag{45}$$

where $\mathbf{A}_i$ and $\mathbf{B}_i$ are given by (35) and (44) with $\mathbf{\Phi} = E[\tilde{\mathbf{h}}\tilde{\mathbf{h}}^H]$, respectively, and ϵ is a non-negative variable. Given ϵ, we can translate this problem into a similar formula as (28) and can solve it using an SDP solver and the randomization procedure. If ϵ is too large, the performance becomes unsatisfactory. On the contrary, too small a value of ϵ makes

	Modified Method
1	Choose ϵ_{up}, ϵ_{low} and ϵ_{dif}.
2	Set $\epsilon = \frac{\epsilon_{\text{up}}+\epsilon_{\text{low}}}{2}$.
3	Solve (45) with ϵ.
4	If (45) is feasible, set $\epsilon_{\text{up}} = \epsilon$. Otherwise, set $\epsilon_{\text{low}} = \epsilon$.
5	Go to 2 unless $\epsilon_{\text{up}} - \epsilon_{\text{low}} \leq \epsilon_{\text{dif}}$.

Table 1 Simulation parameters.

Modulation scheme	QPSK
Number of subcarriers: N	64
CP length: P	16
Channel order: M	12
TEQ length: L	12
Number of antennas: D	2
Number of relays: R	2
Decision delay: δ	0
FEQ	zero-forcing
SNR at the relay: SNR_R	30dB
SDP solver	SeDuMi and YALMIP
Randomization number: N_p	10^2
Bisection parameters: $(\epsilon_{\text{up}}, \epsilon_{\text{low}}, \epsilon_{\text{dif}})$	$(1, 0, 10^{-3})$

the problem (45) infeasible (i.e., no solution can meet the constraints). Thus, the choice of ϵ has a great impact on the system performance. Here, we propose a bisection-based algorithm which can determine ϵ iteratively. The resulting algorithm is summarized in Modified Method.

5. Simulation Results

5.1 Simulation Conditions

We conducted simulations to evaluate the performance of the proposed method. Unless otherwise stated, we used parameters listed in Table 1. The channel tap coefficients $\{h_{r,k}^{(d)}\}, \{\tilde{h}_{r,k}\}$ were modeled as zero-mean complex Gaussian random variables with variance σ_h^2. BER was obtained by averaging over 10^4 simulation trials where each trial has a different channel and different $B = 100$ data blocks. The received SNR at a relay is given by

$$\text{SNR}_\text{R} = \frac{DR\sigma_s^2(M+1)\sigma_h^2}{\sigma_n^2} \tag{46}$$

where σ_n^2 is the noise power at each of the receive antenna of the relay. The SNR at the receiver is given by

$$\text{SNR}_\text{D} = \frac{P_{\text{RT}}(M+1)\sigma_h^2}{\sigma_{\tilde{n}}^2} \tag{47}$$

where $\sigma_{\tilde{n}}^2$ is the noise power at the receiver and $P_{\text{RT}} = \sum_{r=1}^{R} E\left[\left|y'_{r,k}\right|^2\right]$ is the total transmission power at relays. We set the randomization number $N_p = 100$ since the performance hardly changes for $N_p \geq 100$ in our preliminary simulation.

5.2 Filter Design with Perfect CSI

We show the simulation results of the proposed method in (28) when the perfect CSI is available. First, an example of the impulse response of the total channel obtained by the proposed method is shown in Fig. 2. Although the impulse response length of the total channel can be $L+2M = 36$ that is longer than the CP length $P = 16$, the length of non-zero taps is successfully shortened to within the CP length, and IBI can be suppressed perfectly.

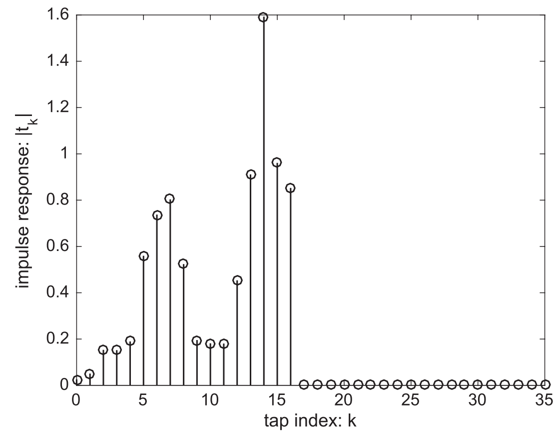

Fig. 2 Example of the impulse response of a total channel.

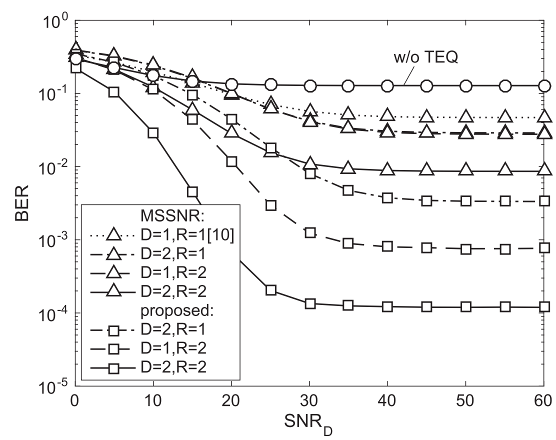

Fig. 3 BER comparison of the conventional method and the proposed method.

Next, we compare the BER performance of the conventional method and that of the proposed method in Fig. 3. The performance of the system without TEQ, where the received signal at relays is just forwarded to the receiver, is poor because it cannot compensate for IBI at all. For the conventional MSSNR-based method with a single relay with a single antenna ($D = 1, R = 1$) [10], the performance is limited due to the residual IBI. Interestingly, its performance can be improved by increasing the numbers of relays R and antennas D, but it is still unsatisfactory because it cannot compensate for deep nulls in the frequency domain. The performance improvement by the proposed method with $D = 2, R = 1$ compared to the MSSNR-based method with $D = 1, R = 1$ is mainly brought by the effect of IBI suppression. Also, the performance of the proposed method further improves by increasing the number of relays from $R = 1$ to $R = 2$ due to mainly the effect of spatial diversity.

To clarify the null cancellation capability of the proposed method, the cumulative distribution function of the worst subcarrier power ($\min_i P_i^{\text{desired}}$) is shown in Fig. 4. It is clearly observed that the worst subcarrier power of the proposed method is larger than that of MSSNR. The result

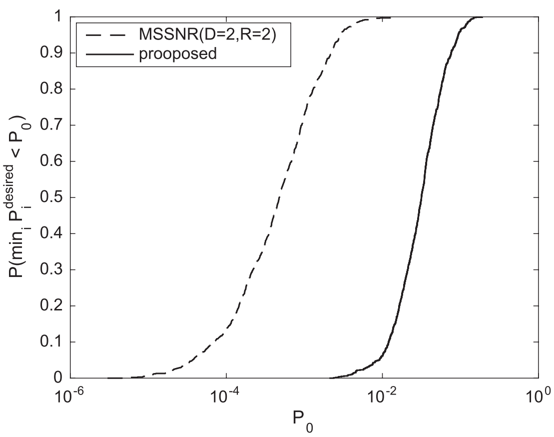

Fig. 4 CDF of the worst subcarrier power.

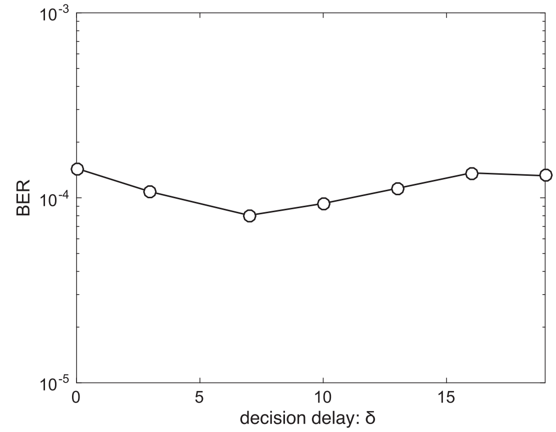

Fig. 6 Influence of a decision delay δ.

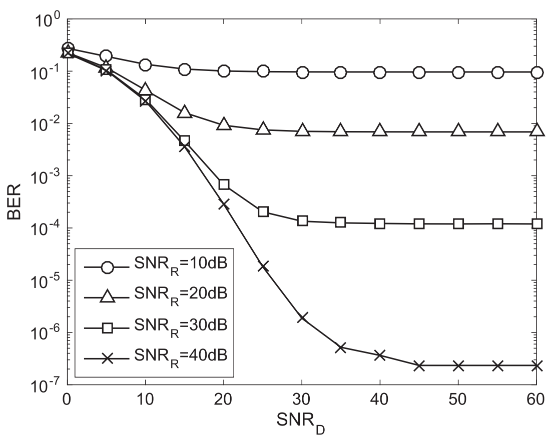

Fig. 5 Effect of received SNR at a relay.

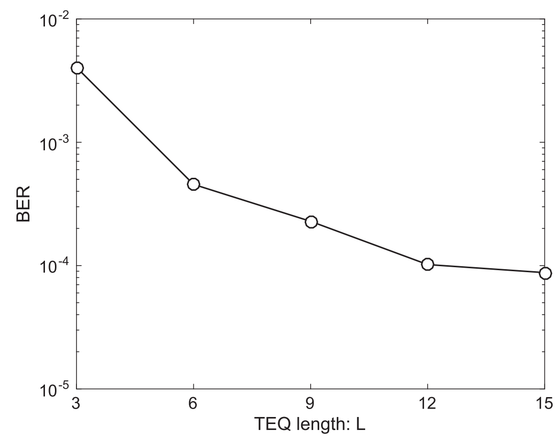

Fig. 7 Influence of TEQ length L.

implies that the proposed method can suppress the effect of deep nulls.

In the following, we show the effect of parameters on the BER performance of the proposed method. In Fig. 5, the effect of the received SNR at a relay is shown. The presence of BER floor is due to the noise at relays. It can be seen that the BER floor becomes lower as the received SNR at the relay increases. Fig. 6 show the influence of a decision delay δ. A moderate value of δ provides the best performance though the difference is slight. Fig. 7 shows the BER performance for various TEQ length, and illustrates that the performance improves as the TEQ length increases. This is because of the increase of degrees-of-freedom. Fig. 8 shows the effect of the number of relays and that of receive antennas at a relay. We can observe that increasing the number of relays is more effective than that of receive antennas. This is due to the additional spatial diversity gain introduced by using multiple relays.

5.3 Filter Design with Partial CSI

Finally, we show the effectiveness of the modified method in (45) when only the channel SOS is available. We adopted the following model for relay-receiver channels [17]:

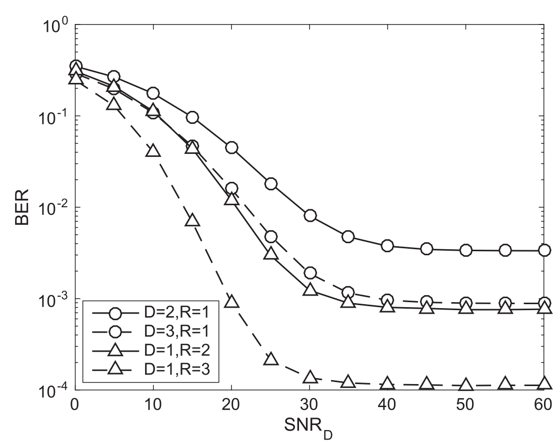

Fig. 8 Effect of the number of relays and receive antennas.

$$\tilde{h}_{r,k} = \sqrt{\frac{1}{1+\alpha}}\bar{h}_{r,k} + \sqrt{\frac{\alpha}{1+\alpha}}h'_{r,k} \tag{48}$$

where $\bar{h}_{r,k}$ and $h'_{r,k}$ are the mean and variable components of the complex channel gain, respectively. The parameter α controls the correlation among the channel taps. For a given channel mean $\bar{\mathbf{h}}$, the correlation matrix of $\tilde{\mathbf{h}}$ is given by

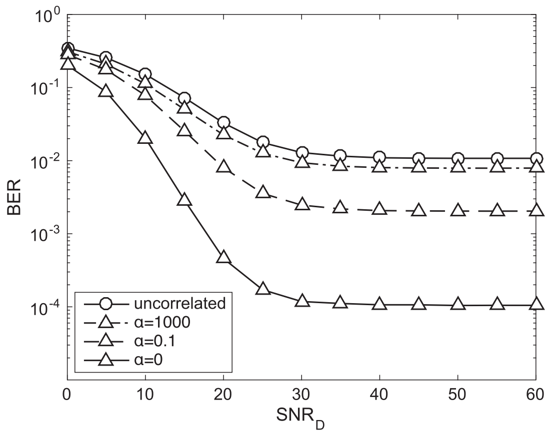

Fig. 9 Influence of the channel correlation.

$$\mathbf{\Phi} = E[\tilde{\mathbf{h}}\tilde{\mathbf{h}}^H] = \frac{1}{1+\alpha}\left(\bar{\mathbf{h}}\bar{\mathbf{h}}^H + \alpha \mathbf{I}_{(M+1)R}\right) \tag{49}$$

where $\bar{\mathbf{h}} = [\bar{\mathbf{h}}_1^T \cdots \bar{\mathbf{h}}_R^T]^T$ and $\bar{\mathbf{h}}_r = [\bar{h}_{r,0} \cdots \bar{h}_{r,M}]^T$. In the simulation, the mean $\bar{h}_{r,k}$ was generated according to $\mathcal{CN}(0, 1)$ only once per simulation trial, and 100 realizations of $h'_{r,k}$ were generated according to $\mathcal{CN}(0, 1)$ for each $\bar{h}_{r,k}$. BER was obtained by averaging over 100 simulation trials.

The BER performance for various α is shown in Fig. 9. The result labeled "uncorrelated" corresponds to the case where $\tilde{\mathbf{h}}$ is uncorrelated with each other, i.e., $E[\tilde{\mathbf{h}}\tilde{\mathbf{h}}^H] = \mathbf{I}$. The performance improves as α decreases, i.e., the channel correlation becomes larger. The modified method can make use of the correlation to improve the system performance. We can also observe that the performance of the modified method in the case of $\alpha = 0$ is almost the same as that of the proposed method with the perfect CSI shown in Fig. 3.

6. Conclusion

In this paper, we proposed a filter design method for filter-and-forward relay networks experiencing IBI. We considered a max-min problem subject to IBI nulling constraint. It was shown that using multiple relays with multiple antennas results in not only complete IBI cancelation but also BER performance improvement due to spatial diversity. Moreover, the effectiveness of the modified method using the partial CSI was confirmed.

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Satoshi Nagai received the B.Eng. and M.Eng. degrees in electrical and electronic engineering from Ibaraki University, Hitachi, Japan, in 2014 and 2016, respectively. In 2016, he joined Hitachi Ltd., Tokyo, Japan. His research interests include signal processing for wireless communications.

Teruyuki Miyajima received the B.Eng., M.Eng., and Ph.D. degrees in electrical engineering from Saitama University, Saitama, Japan, in 1989, 1991, and 1994, respectively. In 1994, he joined Ibaraki University, Hitachi, Japan, where he is currently a professor in the Department of Electrical and Electronic Engineering. His current interests are in signal processing for wireless communications. Dr. Miyajima is a member of IEEE.