

Performance Improvement Scheme for Chaotic Synchronization Based Multiplex Communication Systems

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SUMMARY This paper proposes a method of improving demodulation performance for chaotic synchronization based multiplex communication systems. In a conventional system, the number of data demodulated correctly is limited because transmitted chaotic signals interfere with each other. The proposed system uses a generalized inverse of a matrix formed from chaotic signals at the transmitter. Since this completely cancels the interference between chaotic signals, demodulation performance is greatly improved. The proposed system has the following features: A simple correlation receiver suitable for small terminals can be used; The magnitude of the correlator output is constant for binary data transmission; Analog information data can also be transmitted. Two methods to reduce the peak-to-average power ratio of the transmitted signal are presented.

key words: chaotic communications, chaotic synchronization, security, multiplex communication, generalized inverse

1. Introduction

In the past decade, there has been tremendous interest in the possibilities of exploiting chaos in communications systems [2]–[6]. Due to its wideband characteristics and noise-like behavior, a chaos-based communications system can be expected to be more interference resistant and secure than conventional communications systems. Among the communication systems proposed so far, digital data transmission systems using chaos synchronization phenomena [1] are interesting for both scientific and practical purposes. Recently, some researches have tried to apply chaotic-synchronization-based communication systems to various practical scenarios such as communications in the presence of inter-symbol interference or multiplex communications [7]–[13]. Multiplex communications are important for effective use of bandwidth.

Several multiplexing systems using chaos synchronization have been developed [9]–[13]. In [9], Torikai et al. proposed a method based on time division multiplex where information is encoded in chaotic pulse sequences. This method requires an unrealistically broad bandwidth to send extremely narrow pulses. The method proposed in [10] transmits multiple data by modulating the parameters of chaotic systems and demodulates the data by estimating the

parameters using an extended Kalman filter whose computational complexity is extremely high. Multiplex communications based on code-division multiplexing have also been proposed [11]–[13]. The method in [11] demodulates the information data with the least-squares method, which requires high computational complexity. In addition, this method requires two communication channels for data transmission and synchronization purposes. Inoue et al. [13] proposed a method where both data transmission and synchronization can be done through a single channel. However, its disadvantage is high receiver complexity due to the use of the least-squares method. On the other hand, the method in [12] uses a single channel for both data transmission and synchronization and employs a simple correlation detector. This simplicity is especially desirable for the small terminals of cellular phone or wireless LAN systems. In this system, if the cross-correlation between transmitted chaotic signals is zero, all the data can be perfectly demodulated. However, since the cross-correlation is not always zero, the transmitted chaotic signals interfere with each other. As a result, its demodulation performance, i.e., demodulation accuracy is degraded as the number of multiplexed data increases. Moreover, since fluctuation of the magnitude of the correlator outputs acts as noise when the chaotic sequence length per data is not long, the performance is not satisfactory even if the number of multiplexed data is small.

This paper proposes a preprocessing technique at the transmitter to improve the demodulation performance of chaotic-synchronization-based multiplex communication systems. This can be accomplished by using a generalized inverse of a matrix formed from chaotic signals. Since the interference can be canceled completely, demodulation performance is greatly improved. And a simple correlation receiver suitable for small terminals can be used. Unlike the method in [12], the magnitude of the correlator outputs do not fluctuate. The performance of this system does not depend on either the number of multiplexed data or the chaotic sequence length per data. Moreover, analog information data can be transmitted as well as digital data.

In the next section, we explain our novel chaotic-synchronization-based multiplex communications system. The system's characteristics are examined in Sect. 3, and a peak-to-average power ratio problem [19] of the transmitted signal is dealt with in Sect. 4. Section 5 is a brief conclusion.

Manuscript received December 20, 2003.

Manuscript revised March 31, 2004.

Final manuscript received May 10, 2004.

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2. Chaotic Multiplex Communication

2.1 Proposed System

The block diagram of the proposed system is shown in Fig. 1. In this system, n -multiple information data $\mathbf{b}(i) = [b_1(i) \ b_2(i) \ \cdots \ b_n(i)]^T$ are transmitted in the i -th block. For simplicity, the number of multiplexed data n is assumed to be equal to the order of the chaos systems. As a first step, the fundamental performance limitation in the absence of the channel noise should be clarified. Thus, in this paper, it is assumed that the channel noise can be neglected.

The following chaotic system is prepared in the transmitter:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + F(s(k)) \quad (1)$$

where $\mathbf{x}(k) \in R^{n \times 1}$ is the state of chaotic systems at time k , $\mathbf{A} \in R^{n \times n}$ is a constant matrix whose absolute values of all eigenvalues are smaller than 1, $F(s(k)) = [F_1(s(k)) \ F_2(s(k)) \ \cdots \ F_n(s(k))]^T$, $F_j: R^1 \rightarrow R^1$ is a nonlinear mapping in which each element of $\mathbf{x}(k)$ has chaotic behavior, and $s(k) \in R^1$ is the transmitted signal. The chaotic signal vectors $\mathbf{x}(k)$ form $L \times n$ matrix

$$\mathbf{X}(i) = \begin{bmatrix} \mathbf{x}(iL)^T \\ \mathbf{x}(iL+1)^T \\ \vdots \\ \mathbf{x}((i+1)L-1)^T \end{bmatrix}. \quad (2)$$

The data block $\mathbf{b}(i)$ is transmitted by using chaotic signals of sequence length L . Accordingly, the transmitted signal vector $\mathbf{s}(i)$ can be generated as follows:

$$\mathbf{s}(i) = \begin{bmatrix} s(iL) \\ s(iL+1) \\ \vdots \\ s((i+1)L-1) \end{bmatrix} \quad (3)$$

$$= G \cdot (\mathbf{X}^T(i-1))^+ \mathbf{b}(i) \quad (4)$$

where $G > 0$ is transmission gain, $(\mathbf{X}^T(i))^+$ is generalized

inverse of $\mathbf{X}^T(i)$, which becomes $\mathbf{X}^T(i) \cdot (\mathbf{X}^T(i))^+ = \mathbf{I} \in R^{n \times n}$. Note that the data $\mathbf{b}(i)$ is multiplied by the chaotic signal matrix $(\mathbf{X}^T(i-1))^+$ of the previous block to generate $\mathbf{s}(i)$. The signal $s(k)$, $k = iL, iL+1, \dots, (i+1)L-1$ is transmitted in serial.

The following chaotic system is prepared in the receiver:

$$\mathbf{y}(k+1) = \mathbf{A}\mathbf{y}(k) + F(r(k)) \quad (5)$$

where $\mathbf{y}(k) \in R^n$ is the state of the chaotic systems at time k and $r(k) \in R^1$ is the received signal. The received signal $r(k)$ is equal to $s(k)$ when channel noise is ignored. The error between $\mathbf{x}$ and $\mathbf{y}$ is thus

$$\mathbf{e}(k) = \mathbf{y}(k) - \mathbf{x}(k) \quad (6)$$

$$= \mathbf{A}(\mathbf{y}(k-1) - \mathbf{x}(k-1)). \quad (7)$$

Since absolute values of all eigenvalues of $\mathbf{A}$ are smaller than 1, chaotic synchronization is achieved as follows:

$$\lim_{k \rightarrow \infty} \mathbf{e}(k) = \lim_{k \rightarrow \infty} \mathbf{A}\mathbf{e}(k-1) \quad (8)$$

$$= 0. \quad (9)$$

The correlation detection is used to demodulate the information data. An $L \times n$ matrix can be formed from $\mathbf{y}(k)$ as follows:

$$\mathbf{Y}(i) = \begin{bmatrix} \mathbf{y}(iL)^T \\ \mathbf{y}(iL+1)^T \\ \vdots \\ \mathbf{y}((i+1)L-1)^T \end{bmatrix}. \quad (10)$$

Then, the correlator output vector $\mathbf{v}(i) = [v_1(i) \ v_2(i) \ \cdots \ v_n(i)]^T$ is

$$\mathbf{v}(i) = \frac{1}{G} \mathbf{Y}^T(i-1) \mathbf{r}(i) \quad (11)$$

where $\mathbf{r}(i)$ is the received signal vector given by

$$\mathbf{r}(i) = \begin{bmatrix} r(iL) \\ r(iL+1) \\ \vdots \\ r((i+1)L-1) \end{bmatrix} \quad (12)$$

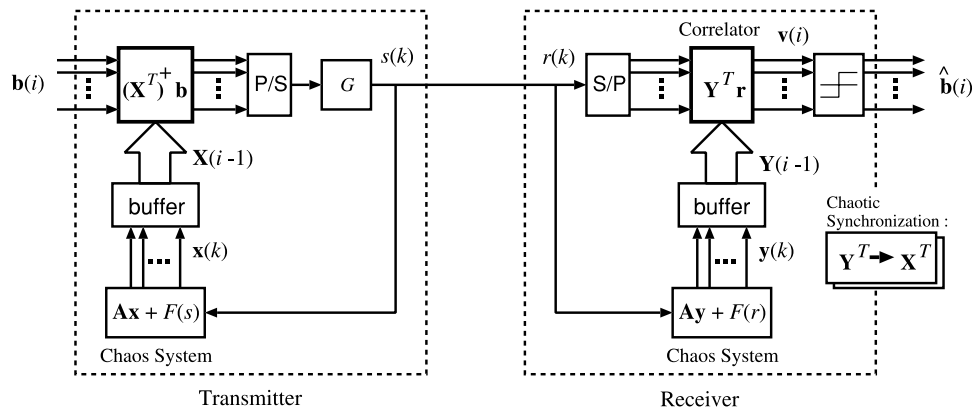


Fig. 1 Block diagram of proposed system.

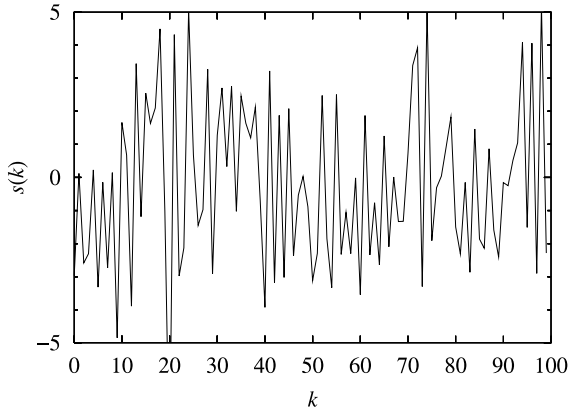


Fig. 2 Transmitted signal $s(k)$ in case of Hénon map. $L = 10$.

When $\mathbf{Y}(i)$ synchronizes with $\mathbf{X}(i)$, the correlator outputs become

$$\begin{aligned} \mathbf{v}(i) &= \mathbf{X}^T(i-1)(\mathbf{X}^T(i-1))^+ \mathbf{b}(i) \\ &= \mathbf{b}(i). \end{aligned} \quad (13)$$

Clearly, the output $\mathbf{v}(i)$ contains no interference component and is exactly equal to the data. In the binary data case, the magnitude of the correlator output is constant if synchronization is achieved unlike in the conventional system [12]. In this case, to eliminate the effect of the synchronization error, the data is estimated as follows:

$$\hat{\mathbf{b}}(i) = \text{sgn}(\mathbf{v}(i)) \quad (14)$$

where $\text{sgn}(\mathbf{x}) : R^n \rightarrow R^n$ is a function which returns the sign of each element of $\mathbf{x}$.

2.2 Computer Simulations

Computer simulations using a Hénon map and Global coupled map(GCM) were carried out to confirm the effectiveness of the proposed system. The initial state of $\mathbf{s}(0)$ is given by a uniform random number within $(-1, +1)$.

(a) Hénon map:

$$\begin{aligned} x_1(k+1) &= x_2(k) + 1 - 1.4s(k)^2, \\ x_2(k+1) &= 0.3x_1(k). \end{aligned}$$

The constant matrix $\mathbf{A}$ and map F are

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 0.3 & 0 \end{bmatrix},$$

$$F(s(k)) = \begin{bmatrix} 1 - 1.4s(k)^2 \\ 0 \end{bmatrix},$$

$n = 2$, $L = 10$, and $G = 5$. Figures 2 and 3 show the transmitted signal $s(k)$ and squared synchronization errors $e_1^2(k)$ and $e_2^2(k)$. It can be seen that the behavior of $s(k)$ looks like random noise. From Fig. 3, it is clear that the synchronization error has converged to zero and synchronization has been achieved. In Fig. 4, the

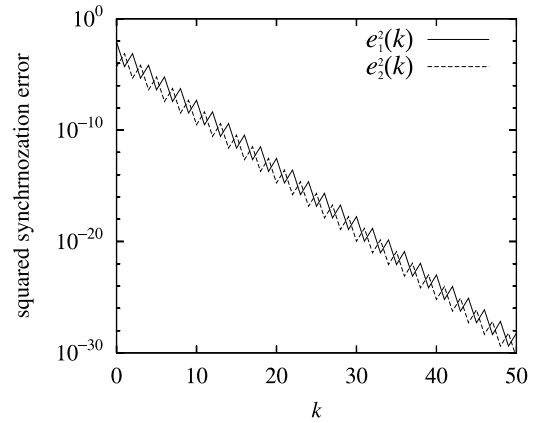
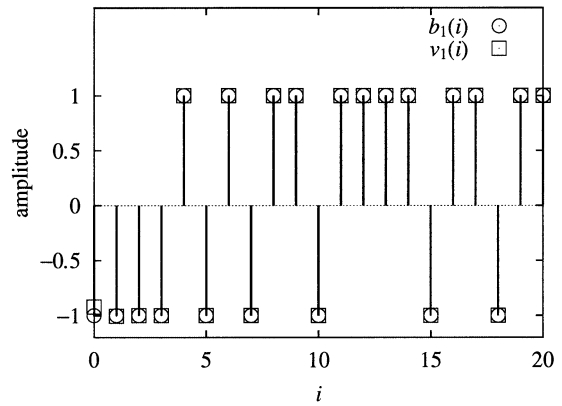
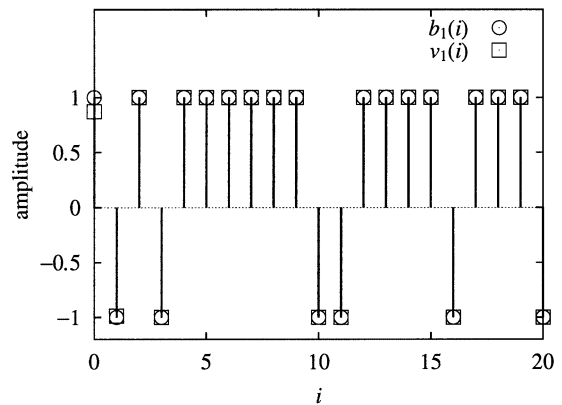


Fig. 3 Squared synchronization error $e_1^2(k)$ and $e_2^2(k)$ in case of Hénon map.



(a) $b_1(i)$, $v_1(i)$



(b) $b_2(i)$, $v_2(i)$

Fig. 4 Information data $b_1(i)$, $b_2(i)$ and correlator output $v_1(i)$, $v_2(i)$ in case of Hénon map.

information data and correlator outputs are shown. At only the first bit, error between data and correlator output can be observed. The other information data can be accurately estimated.

(b) GCM: The parameters are $n = 5$, $L = 100$, $G = 5$,

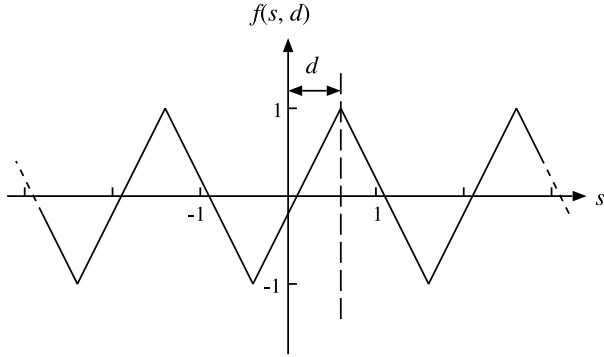
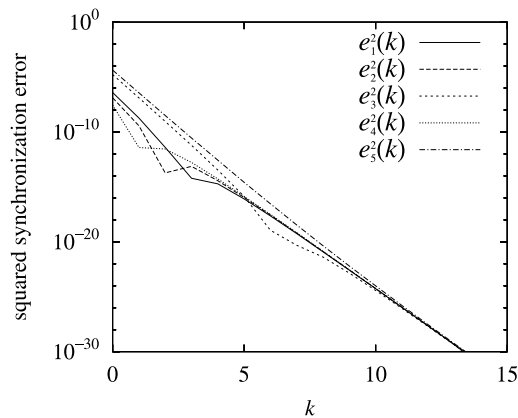
Fig. 5 Map $f(s, d)$.

Fig. 6 Synchronization square error in GCM.

$$\mathbf{A} = \begin{bmatrix} 0.1 & 0.01 & 0.01 & 0.01 & 0.01 \\ 0.01 & 0.1 & 0.01 & 0.01 & 0.01 \\ 0.01 & 0.01 & 0.1 & 0.01 & 0.01 \\ 0.01 & 0.01 & 0.01 & 0.1 & 0.01 \\ 0.01 & 0.01 & 0.01 & 0.01 & 0.1 \end{bmatrix},$$

and $F(s(k)) = [f(s(k), -0.45) \ f(s(k), -0.27) \ f(s(k), -0.09) \ f(s(k), 0.09) \ f(s(k), 0.27)]^T$, where f is as shown in Fig. 5. Figure 6 shows the squared synchronization errors. From this figure, it is seen that synchronization is achieved quickly. Although it is not shown here, we have confirmed that the information data can be demodulated successfully.

2.3 Conventional System

A conventional system [12] is explained briefly. The following chaotic system is prepared in the transmitter:

$$x_j(k+1) = a_j x_j(k) + F_j(s(k)), \quad j = 1, \dots, n \quad (15)$$

$$s(k) = \sum_{j=1}^n b_j(k) x_j(k) \quad (16)$$

where $x_j(k) \in R^1$ is the j -th state of the chaotic system at time k , $a_j \in R^1$ is a constant whose absolute value is smaller than 1, and $F_j : R^1 \rightarrow R^1$ is a nonlinear mapping in which

$x_j(k)$ has chaotic behavior. The following chaotic system is prepared in the receiver:

$$y_j(k+1) = a_j y_j(k) + F_j(r(k)), \quad (17)$$

where $y_j(k) \in R^1$ is the j -th state of the chaotic system at time k . The correlator output $v_j(i)$ is

$$v_j(i) = \sum_{k=(i-1)L}^{iL-1} s(k) y_j(k) \quad (18)$$

$$= \sum_{k=(i-1)L}^{iL-1} \sum_{l=1}^n b_l(k) x_l(k) y_j(k). \quad (19)$$

The chaotic systems in (15) and (17) can also be represented as a matrix form in Eq. (1) and Eq. (5), respectively. The transmitted signal of the conventional system can also be written as a matrix form as follows:

$$\mathbf{s}(i) = \mathbf{X}(i) \mathbf{b}(i). \quad (20)$$

When the channel noise is ignored, the correlator output in achieving chaotic synchronization becomes

$$\mathbf{v}(i) = \frac{1}{L} \mathbf{X}^T(i) \mathbf{X}(i) \mathbf{b}(i). \quad (21)$$

3. Characteristics of Proposed System

3.1 Demodulation Performance

The proposed system is compared with a conventional system [12] in terms of demodulation performance, i.e., how accurately the multiplexed data can be demodulated. Firstly, let us clarify the reason of the performance degradation of the conventional chaos-based multiplex communication systems described in 2.3. Let us consider the first element $v_1(i)$ in (18), which is the first correlator's output. This can be divided into

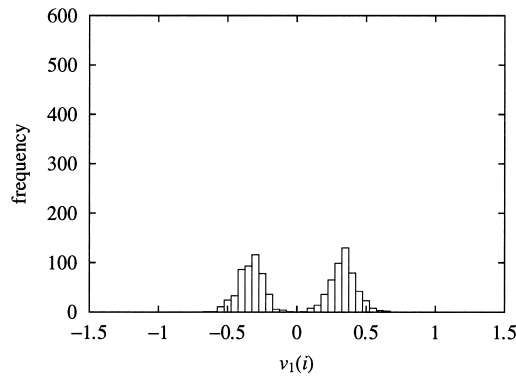
$$v_1(i) = b_1(i) v_{d1}(i) + \sum_{j=2}^n b_j(i) v_{e1j}(i) \quad (22)$$

where

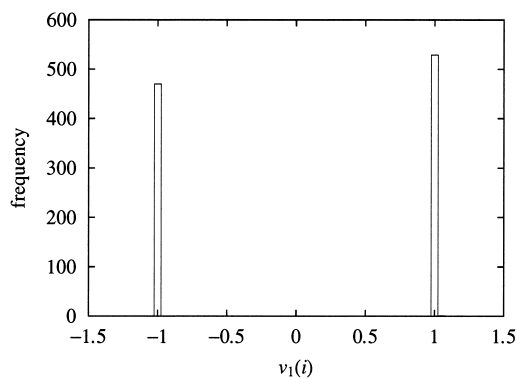
$$v_{d1}(i) = \frac{1}{L} \sum_{k=iL}^{(i+1)L-1} x_1^2(k), \quad (23)$$

$$v_{e1j}(i) = \frac{1}{L} \sum_{k=iL}^{(i+1)L-1} x_1(k) x_j(k). \quad (24)$$

The first term of Eq. (22) is the information data component, and the second term is an interference component. There are two causes limiting the demodulation performance of the conventional system. The one is that the interference represented by the second term becomes non-zero. The interference increases in proportion to the number of multiplexed data n ; thus the data can not be exactly demodulated from Eq. (22). The other cause is that the data component of the



(a) Conventional system



(b) Proposed system

Fig. 7 Histograms of correlator output $v_1(i)$.

first term fluctuates because $v_{d1}(i)$ varies every data. The fluctuation acts as noise. The fluctuation increases when L is small, even if n is small.

On the other hand, it is clear from Eq. (13) that there is no interference or fluctuation in the proposed system. Figure 7 shows histograms of $v_1(i)$ of the conventional and proposed systems for 1000 samples in which the GCM is used and the parameters are the same as in Sect. 2.2(b). The correlator output of the conventional system fluctuates (Fig. 7(a)), whereas the correlator output of the proposed system becomes -1 or $+1$ and is of constant magnitude (Fig. 7(b)). Figure 8 shows the information data $b_1(i)$ and the correlator output $v_1(i)$ of the conventional system for $n = 5$. Some bit errors (indicated by “ $\times$ ” in the figure) occurred.

To evaluate the demodulation performance, the normalized demodulation error is defined as

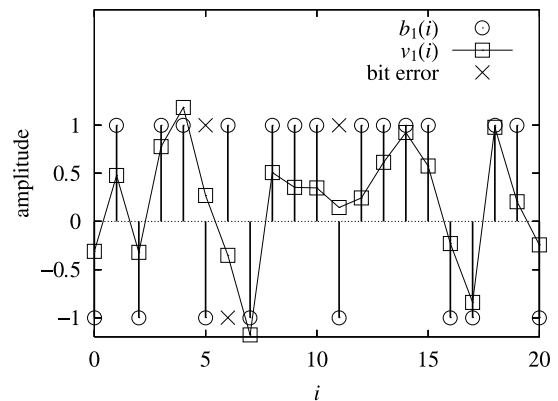
$$\varepsilon_1(n) = \overline{\left(b_1(i) - \frac{v_1(i)}{m_{d1}(i)}\right)^2} \quad (25)$$

$$= \frac{1}{m_{d1}^2} \left(\overline{(v_{d1}(i) - m_{d1})^2} + \sum_{j=2}^n \sigma_{e1j}^2 \right) \quad (26)$$

where over-line means time average and

$$m_{d1} = \overline{v_{d1}(i)}, \quad (27)$$

$$\sigma_{e1j}^2 = \overline{v_{e1j}(i)^2}. \quad (28)$$

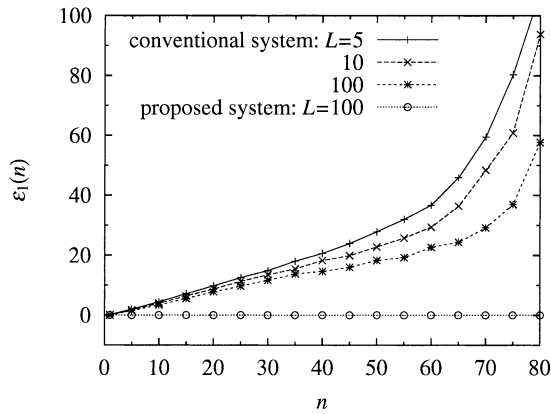
**Fig. 8** Information data $b_1(i)$ and correlator output $v_1(i)$ of the conventional system. $n = 5$.

The first term in the parenthesis of Eq. (26) is the fluctuation of the desired data component $v_{d1}(i)$ in the correlator output $v_1(i)$, and the second term is the square average of the interference component. If there is no interference or fluctuation, the demodulation error $\varepsilon_1(n)$ becomes zero. Clearly, the demodulation error $\varepsilon_1(n)$ is zero for the proposed system. In the conventional system [12], though σ_{e1j}^2 is very small, but it is non-zero. From Eq. (26), therefore, it can be seen that the demodulation error $\varepsilon_1(n)$ increases, when the number of multiplexed data n is increased. What is worse, $\varepsilon_1(n)$ can not be zero even if $n = 1$, since $v_{d1}(i)$ fluctuates every block i . Figure 9(a) shows the $\varepsilon_1(n)$ s of the conventional and proposed system, where the GCM is used and the other parameters are the same as in Sect. 2.2(b) (except n and L). In the conventional system, $\varepsilon_1(n)$ increases as n increases. Moreover, $\varepsilon_1(n)$ is degraded when L is small. A similar result using a different chaos in [12] is obtained in Fig. 9(b) where the j -th mapping $F_j(x)$ is the j -times iteration of $f(x, 0)$ in Fig. 5. When $n = 5$, this normalized demodulation error $\varepsilon(5)$ is about 1.48 in Fig. 9(a). In this case, some bit errors occurred at the rate of about 20 percent. On the other hand, in the proposed system, $\varepsilon_1(n)$ becomes 0 regardless of n and L .

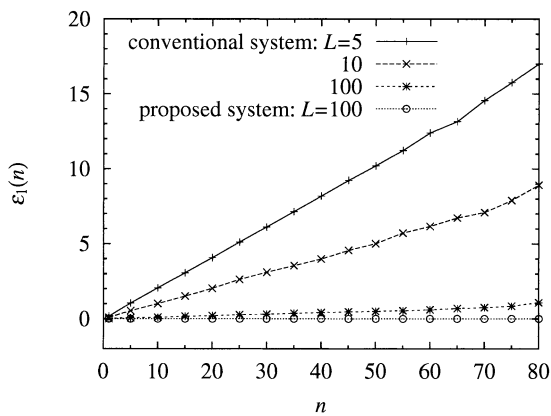
3.2 Transmitting Analog-Valued Data

The conventional system [12] can only transmit digital data by using a threshold decision device because the correlator output is not equal to the data. Thus, analog-valued data can not be transmitted. In contrast, the proposed system can transmit analog-valued data because the correlator output is always equal to the data.

Computer simulation of analog-valued data transmission was carried out. Figure 10 shows the transmitted analog-valued data $b_1(i) = \sin(0.5i) + 0.5 \cos(0.2i)$ and the estimated signal $v_1(i)$. The parameters used are the same as in Sect. 2.2(b). From the figure, it is clear that the analog-valued data can be demodulated exactly. We have confirmed that the proposed system could also exactly demodulate the other analog-valued data $b_2(i), \dots, b_n(i)$.



(a) Map in Fig. 5



(b) Map in [12]

Fig. 9 Demodulation error $\varepsilon_1(n)$ vs. the number of multiplexed data n .

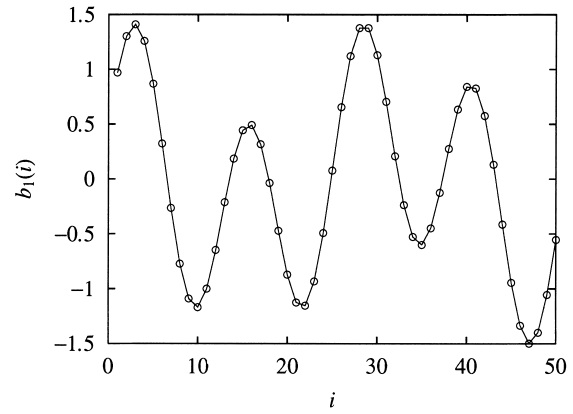
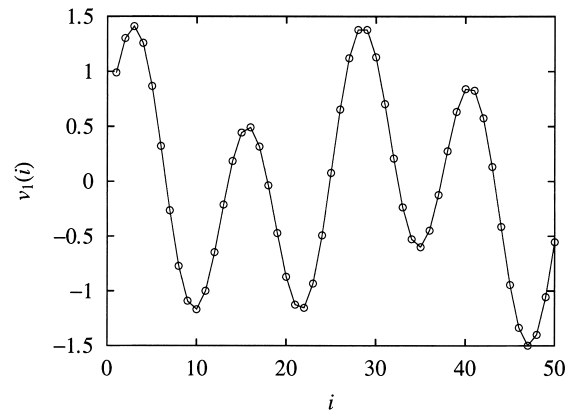
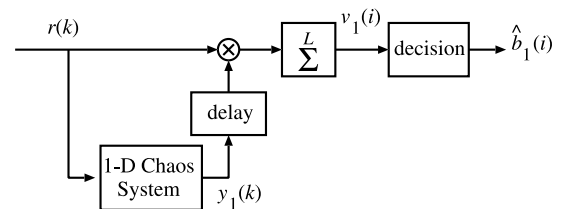
3.3 Complexity of Receiver

As mentioned in Sect. 3.1, a major problem of the conventional system is interference between chaotic signals. The interference can be eliminated if the decorrelator [14] is used in the receiver. However, the receiver becomes complicated in this case. In the proposed system, the receiver has been simplified by complicating the transmitter instead. In a cellular phone system, for example, the base station can be large and complicated, but the mobile terminal should be small. Thus, the proposed system is suitable for such systems.

Interestingly, the receiver of the proposed system is further simplified, if $\mathbf{A}$ is a diagonal matrix as shown in Fig. 11. The receiver in Fig. 11 includes a discrete-time one-dimensional chaotic system as follows:

$$y_1(k+1) = a_1 y_1(k) + f_1(s(k)) \quad (29)$$

where a_1 is a diagonal element of the constant matrix $\mathbf{A}$ and $f_1 : R^1 \rightarrow R^1$ is the map of the first element in the chaos map F . If chaotic synchronization is achieved, the correlator

(a) Information data $b_1(i)$ (b) Correlator output $v_1(i)$ **Fig. 10** Analog-valued data $b_1(i)$ and the correlator output $v_1(i)$.**Fig. 11** Simple receiver for one-channel demodulation in case of diagonal $\mathbf{A}$.

output $v_1(i)$ will be

$$\begin{aligned} v_1(i) &= [y_1((i-1)L) \ y_1((i-1)L+1) \ \cdots \ y_1(iL-1)] \\ &\quad \times \mathbf{r}(i) \\ &= [x_1((i-1)L) \ x_1((i-1)L+1) \ \cdots \ x_1(iL-1)] \\ &\quad \times (\mathbf{X}^T(i-1))^+ \mathbf{b}(i) \\ &= b_1(i). \end{aligned} \quad (30)$$

The output $v_1(i)$ is exactly the same as the data $b_1(i)$. On the other hands, the conventional system using the decorrelator requires n chaotic systems and n correlator outputs to obtain the data estimate without interference, even if $\mathbf{A}$ is a

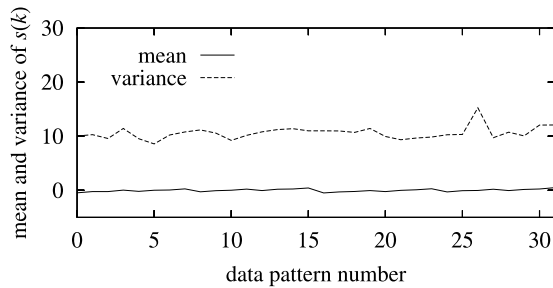


Fig. 12 Mean and variance of transmitted signal $s(k)$.

diagonal matrix. Thus, the structure of the receiver is very complicated.

3.4 Security

Finally, let us briefly mention the security characteristics of the proposed system. In general, the adoption of higher dimensional chaos systems enables us to strengthen the security of the communications system [6], [15]. Since the proposed approach can be applied to chaotic systems of any dimension, the security can be enhanced.

In [16] and [17], Short reported that analog signals transmitted by using the chaotic masking method can be extracted without knowing the chaotic system used in the transmitter. On the other hand, as far as we know, there is very little research on attack methods against the other types of chaotic communications systems for digital data transmissions including chaos shift keying [18]. The idea behind an attack against digital data transmissions is that if the received signals' characteristics, such as statistics, e.g., mean or variance, or reconstructed return maps, are different for every data, the data can be extracted by finding the difference in the characteristics.

Clearly, in the proposed systems, reconstructed return maps are not useful for this purpose because the return maps become very complex due to the high dimensionality of chaos that has been used. If one tries to evaluate statistics in multiple communication systems, 2^n statistics have to be estimated for binary transmissions. This is computationally expensive for large n . Moreover, if the maps $F(\cdot)$ are chosen carefully, the statistics do not depend on the data. As an example, Fig. 12 shows the mean and variance of the received signals in the case of binary transmission with $n = 5$ sources, where the map in Fig. 5 has been used. There are $2^5 = 32$ data patterns. No meaningful dependence on the mean and variance of the data pattern can be found. Therefore, the proposed approach has the possibility of providing secure communication systems.

4. Reduction of Peak-to-Average Power Ratio

One drawback of the proposed system is that the transmitted signal tends to have a large peak-to-average power ratio (PAPR) which is defined as the peak power in a time period versus the average power in the same period. The PAPR

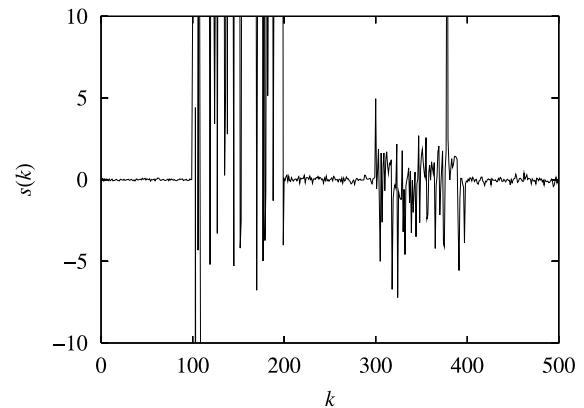


Fig. 13 Transmitted signal $s(k)$ at $G = 1$, $L = 100$.

of the transmitted signal should be low since the signal deteriorates because of the nonlinear characteristics of power amplifiers [19]. In the proposed system, the transmitted signal $s(k)$ may take very small values and very large values in turn. Figure 13 shows an example of such a transmitted signal $s(k)$. Here $G = 1$, and the global coupled map in Sect. 2.2(b) is used. As a result of such vibration, the transmitted signal has a large PAPR. The remainder of the section examines how to deal with the PAPR issue.

The transmitted signal $\mathbf{s}(i)$ is generated using the generalized inverse of $\mathbf{X}^T(i-1)$. The norm of $\mathbf{s}(i)$ tends to decrease. For example, the norm becomes a minimum when the Moore-Penrose inverse is used. If the norm of $\mathbf{s}(i)$ is small, only a narrow domain of the chaos map F is used. Then, the columns of $\mathbf{X}^T(i)$ will not be linearly independent, then the condition number of $\mathbf{X}^T(i)$ is large. As a result, the norm of the transmitted signal $\mathbf{s}(i+1)$ generated according to the generalized inverse of $\mathbf{X}^T(i)$ increases. In the next block, conversely, the whole domain of F is used, because the norm of $\mathbf{s}(i+1)$ is large. The linear independence on the columns of $\mathbf{X}^T(i+1)$ rises. Thus, using the generalized inverse of $\mathbf{X}^T(i+1)$ decreases the norm of $\mathbf{s}(i+2)$. These two situations appear in turn.

The small norm of $\mathbf{s}(i)$ is feedback into the chaotic system. Hence, the undesired vibration will occur. Two methods can be used to increase the norm of $\mathbf{s}(i)$:

I) Adjustment of coefficient matrix $\mathbf{A}$

Small correlations of each chaotic signal lead to a transmitted signal $\mathbf{s}(i)$ with a small norm. Thus, the non-diagonal element of $\mathbf{A}$ may be increased to increase correlations of the chaotic signals.

II) Adjustment of transmission gain G

This method directly increases the norm of $\mathbf{s}(i)$ by increasing G to more than 1.

Figure 14 shows the signal $s(k)$ transmitted with the above two methods, where

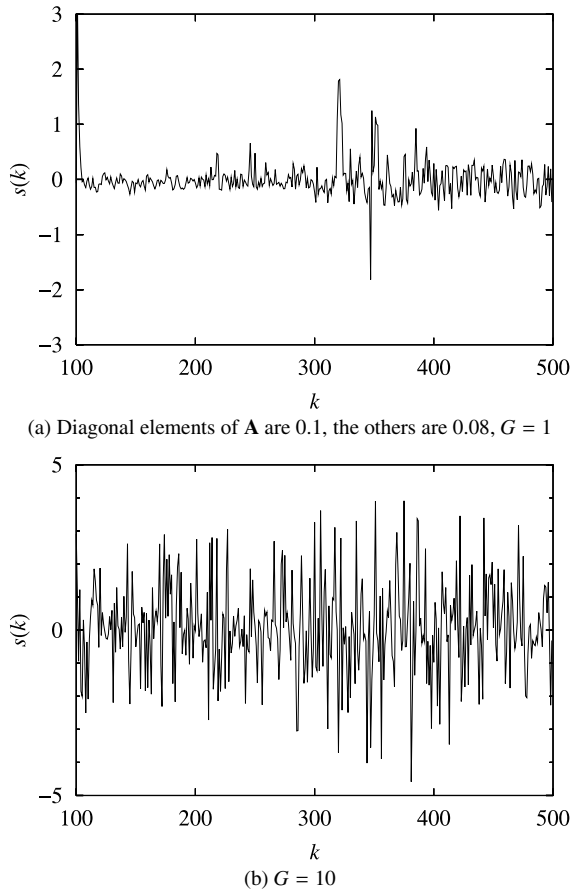


Fig. 14 Transmitted signal $s(k)$.

$$\mathbf{A} = \begin{bmatrix} 0.1 & 0.08 & 0.08 & 0.08 & 0.08 \\ 0.08 & 0.1 & 0.08 & 0.08 & 0.08 \\ 0.08 & 0.08 & 0.1 & 0.08 & 0.08 \\ 0.08 & 0.08 & 0.08 & 0.1 & 0.08 \\ 0.08 & 0.08 & 0.08 & 0.08 & 0.1 \end{bmatrix},$$

in I) and $G = 10$ in II). It is clear that the PAPR has been reduced in comparison with Fig. 13, and either method appears to be effective. Method II) is advantageous in that only the transmission gain G needs adjustment.

5. Conclusions

This paper proposed a method of improving the demodulation performance for chaotic synchronization based multiplex communication systems. The method employs a pre-processing technique at the transmitter using a generalized inverse, which can greatly reduce the demodulation error. A simple correlation receiver which is suitable for small terminals can be used. The magnitude of the correlator output for binary data is constant, unlike the conventional method. We have shown that the performance of the proposed system do not depend on the number of the multiplexed data and the chaos sequence length per data. In addition, we showed that analog information data can be transmitted and that the peak-to-average power ratio problem of the transmitted sig-

nal could be solved.

Channel noise is of importance as it effectively limits the channel capacity. Future work will examine the performance of the proposed system in noisy channels. From the practical point of view, comparison with conventional communication systems in terms of interference resistance and security capabilities should be done in the future.

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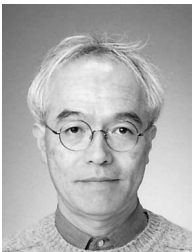
Commun., vol.48, no.10, pp.1755–1764, Oct. 2000.



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