User Fairness in Asynchronous NOMA with Transmit Equalization

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***Abstract*—In this paper, we propose an ANOMA scheme with transmit equalization to achieve user fairness over frequency-selective channels. In this ANOMA scheme, inter-user interference and inter-symbol interference are mitigated by introducing a timing offset (TO) and applying linear precoding with a transmit filter. To ensure fairness, the transmitter is designed based on a max-min fairness optimization framework. Simulation results demonstrate that ANOMA achieves higher rates than conventional NOMA. Moreover, the results show that, for specific TO values, ANOMA without successive interference cancellation (SIC) can outperform ANOMA with SIC in terms of rates.**

***Index Terms*—asynchronous NOMA, user fairness, transmit equalization, frequency-selective channels.**

I. Introduction

The demand for the Internet of Everything (IoE) is expected to increase significantly with the integration of artificial intelligence. To meet this demand, next-generation wireless systems must address several technical challenges [1]. One of the most fundamental challenges is the design of advanced multiple-access techniques. In particular, achieving ultra-dense connectivity of the order of 10^8 devices/km^2 is highly desirable, and many approaches have been proposed to this end. Among them, non-orthogonal multiple access (NOMA), especially its power-domain scheme, has emerged as a strong candidate [2].

In power-domain NOMA (hereafter referred to as NOMA), the base station (BS) transmits downlink signals to multiple user equipment (UE) by multiplexing them at different power levels on the same frequency resource. Inter-user interference (IUI) resulting from this superposition can be mitigated using successive interference cancellation (SIC) [3]. However, the SIC decoding order is typically determined based on channel state information (CSI), which can lead to unfair rate allocation, especially in scenarios where strict user fairness is required [4].

To suppress IUI through an alternative approach, asynchronous NOMA (ANOMA) has been proposed [5]. In the ANOMA downlink, an intentional timing offset (TO) is introduced at the transmitter, causing the received signals to become misaligned at each UE. This misalignment effectively reduces IUI, regardless of the SIC decoding order. To enable ANOMA in wideband communications over frequency-selective channels, a single-carrier ANOMA downlink scheme was proposed in [6], where inter-symbol interference (ISI) is mitigated by applying transmit equalization using finite impulse response (FIR) filters. This scheme simplifies receiver design because ISI suppression is handled on the transmitter side. However, this previous work focused only on the fundamental aspects of ANOMA with transmit equalization and did not consider transmitter optimization. Furthermore, to the best of our knowledge, there has still been no discussion of user fairness in ANOMA.

Motivated by these gaps, in this paper, we propose an ANOMA scheme with transmit equalization to achieve user fairness. In the proposed scheme, transmit filters for all UEs are jointly designed to ensure fairness. The filter optimization problem is formulated to maximize the minimum user rate. Simulation results demonstrate that the proposed ANOMA scheme achieves user fairness at higher rates than the conventional NOMA scheme. More interestingly, owing to the effective suppression of IUI by the intentionally introduced TO, fairness can be achieved without SIC at higher user rates than with SIC.

Throughout this paper, we use the following notations: $(\cdot)^T$, $(\cdot)^H$, and $\|\cdot\|$ denote the transpose, Hermitian transpose, and norm of the vector or matrix, respectively. $\mathbf{I}_N$ denotes the $N \times N$ identity matrix. $\mathbf{0}_{N\times N'}$ denotes the $N \times N'$ zero matrix. $\mathrm{toep}(\mathbf{A}, L)$ denotes the block-Toeplitz matrix defined by $\mathrm{toep}(\mathbf{A}_{N\times N'}, L) \triangleq [\mathbf{A}_0^T\,\mathbf{A}_1^T \cdots \mathbf{A}_L^T]^T$ where $\mathbf{A}_l \triangleq [\mathbf{0}_{N\times l}\,\mathbf{A}_{N\times N'}\,\mathbf{0}_{N\times(L-l)}]$. $\mathrm{diag}(\mathbf{A}_1 \cdots \mathbf{A}_N)$ is a block diagonal matrix with $\mathbf{A}_n$ on diagonal. $\mathrm{Tr}(\mathbf{A})$ indicates the trance of $\mathbf{A}$. $\mathbf{A} \succeq 0$ implies that $\mathbf{A}$ is a semidefinite matrix. $\mathcal{CN}(0, \sigma^2)$ denotes the zero mean circularly symmetric complex Gaussian distribution with variance σ^2. $[\mathbf{A}]_n$ and $(\mathbf{A})_n$ denote the n-th column vector and the matrix obtained by removing n-th column vector of matrix $\mathbf{A}$, respectively. $(\mathbf{a})_n$ denotes the vector obtained by removing the n-th elements of vector $\mathbf{a}$.

II. System Model

We consider an ANOMA downlink with a BS equipped with N_a antennas and two UEs, each equipped with a single antenna, as shown in Fig. 1. We assume that UE1 is closer to the BS than UE2 and uses SIC to cancel the signal intended for UE2. We also assume that each channel between the BS and UE is frequency-selective and modeled as an FIR filter. The BS has perfect CSI for all channels.

The data symbol for UEi at time k is denoted as $s_{i,k}$. We assume that the data symbols $s_{i,k}$ are normalized and i.i.d. sequence. Each antenna at the BS is associated with two FIR filters of length $L_\mathrm{w}+1$, each corresponding to a different UE for interference reduction. The output of the n_a-th filter for UEi whose IR is $\mathbf{w}_{n_\mathrm{a},i} \triangleq [w_{n_\mathrm{a},i,0}\, w_{n_\mathrm{a},i,1} \cdots w_{n_\mathrm{a},i,L_\mathrm{w}}]^T$ is given by

$$x_{n_\mathrm{a},i,k} \triangleq \mathbf{w}_{n_\mathrm{a},i}^H \mathbf{s}_{i,k}, \tag{1}$$

where $\mathbf{s}_{i,k} \triangleq [s_{i,k}\, s_{i,k-1} \cdots s_{i,k-L_\mathrm{w}}]^T$.

Then, a TO τ_i is added to each filter output to reduce IUI. We employ a real-valued pulse of unit energy $p(t)$, which is limited to the interval $[0,T]$. We consider the case $0 \le \tau_1 < \tau_2 < T$ without loss of generality, and define the difference as $\Delta\tau = \tau_2 - \tau_1$. A baseband continuous-time transmit signal is expressed as $x_{n_\mathrm{a},i}(t) = \sum_k x_{n_\mathrm{a},i,k}\, p(t-kT-\tau_i)$. Finally, two signals are multiplexed and transmitted from the n_a-th antenna, represented as $x_{n_\mathrm{a}}(t) = x_{n_\mathrm{a},1}(t) + x_{n_\mathrm{a},2}(t)$.

The signal received at UEi is expressed as $y_i(t) = \sum_{n_\mathrm{a}=1}^{N_\mathrm{a}} \sum_{l=0}^{L_{\mathrm{h},i}} h_{n_\mathrm{a},i,l}\, x_{n_\mathrm{a}}(t-lT) + n_i(t)$ where $h_{n_\mathrm{a},i,l}$ is the l-th coefficient of IR of the channel between the n_a-th antenna of BS and UEi of length $L_{\mathrm{h},i}+1$, and $n_i(t) \sim \mathcal{CN}(0,\sigma_i^2)$ is the additive white Gaussian noise. Correlators process the received continuous-time signal to convert it into a discrete-time signal. Because UE1 performs SIC, it requires two correlators matched to $p(t-kT-\tau_i), i=1,2$, while UE2 requires one correlator matched to $p(t-kT-\tau_2)$. In UEi, the output of the correlator matched to $p(t-kT-\tau_j)$ is given by

$$\begin{aligned} y_{ij,k} &\triangleq \int_{kT+\tau_j}^{(k+1)T+\tau_j} y_i(t)\, p(t-kT-\tau_j)\, dt \\ &= \sum_{n_\mathrm{a}=1}^{N_\mathrm{a}} \sum_{l=0}^{L_{\mathrm{h},i}} h_{n_\mathrm{a},i,l}\, \tilde{x}_{n_\mathrm{a},j,k-l} + n_{ij,k} \end{aligned} \tag{2}$$

where

$$\tilde{x}_{n_\mathrm{a},1,k} \triangleq x_{n_\mathrm{a},1,k} + \left(\xi_{\Delta\tau} x_{n_\mathrm{a},2,k} + \xi_{\Delta\tau-T} x_{n_\mathrm{a},2,k-1}\right), \tag{3}$$

$$\tilde{x}_{n_\mathrm{a},2,k} \triangleq x_{n_\mathrm{a},2,k} + \left(\xi_{\Delta\tau} x_{n_\mathrm{a},1,k} + \xi_{\Delta\tau-T} x_{n_\mathrm{a},1,k+1}\right), \tag{4}$$

$\xi_{\Delta\tau} \triangleq \int_0^T p(t)p(t-\Delta\tau)dt$ is the autocorrelation function of the pulse $p(t)$, and $n_{ij,k}$ is the discrete-time noise. In Eqs. (3) and (4), the second term represents the IUI component, which is distributed over two consecutive symbols due to asynchronous transmission. Because $\xi_{\Delta\tau} \le 1$, the power of the IUI component in ANOMA is smaller than that in NOMA. Substituting Eq. (1) into Eq. (2) yields

$$y_{11,k} = \mathbf{w}_1^H \mathbf{H}_{11,0} \mathbf{s}_{11,k} + \mathbf{w}_2^H \mathbf{H}_{12,\Delta\tau} \mathbf{s}_{12,k-1} + n_{11,k}, \tag{5}$$

$$y_{i2,k} = \mathbf{w}_2^H \mathbf{H}_{i2,0} \mathbf{s}_{i2,k-1} + \mathbf{w}_1^H \mathbf{H}_{i1,\Delta\tau} \mathbf{s}_{i1,k} + n_{i2,k} \tag{6}$$

where $\mathbf{w}_i \triangleq \left[\bar{\mathbf{w}}_{i,0}^T\, \bar{\mathbf{w}}_{i,1}^T \cdots \bar{\mathbf{w}}_{i,L_\mathrm{w}}^T\right]^T$, $\bar{\mathbf{w}}_{i,l} \triangleq [w_{1,i,l}\, w_{2,i,l} \cdots w_{N_\mathrm{a},i,l}]^T$, $\mathbf{s}_{ij,k} \triangleq [s_{j,k+1}\, s_{j,k} \cdots s_{j,k-L_i-1}]^T$, $\mathbf{H}_{ij,\Delta\tau} \triangleq \mathbf{H}_i \boldsymbol{\Xi}_{ij,\Delta\tau}$, $\mathbf{H}_i \triangleq \mathrm{toep}\left([\mathbf{h}_{i,0}\, \mathbf{h}_{i,1} \cdots \mathbf{h}_{i,L_{\mathrm{h},i}}], L_\mathrm{w}\right) \in \mathbb{C}^{N_\mathrm{a}(L_\mathrm{w}+1)\times L_i}$, $\mathbf{h}_{i,l} \triangleq [h_{1,i,l}\, h_{2,i,l} \cdots h_{N_\mathrm{a},i,l}]^T$, $\boldsymbol{\Xi}_{i1,\Delta\tau} \triangleq \mathrm{toep}\left([\xi_{\Delta\tau-T}\, \xi_{\Delta\tau}], L_i-1\right)$, $\boldsymbol{\Xi}_{i2,\Delta\tau} \triangleq \mathrm{toep}\left([\xi_{\Delta\tau}\, \xi_{\Delta\tau-T}], L_i-1\right)$, $\boldsymbol{\Xi}_{ij,\Delta\tau} \in \mathbb{R}^{L_i\times(L_i+1)}$, and $L_i = L_\mathrm{w} + L_{\mathrm{h},i} + 1$. Then, SIC cancels the IUI component in Eq. (5). We assume that the decision delay is $k_\mathrm{d} = \left\lfloor \frac{L_i+1}{2} \right\rfloor$ and that SIC can cancel symbols prior to the k_d-th symbol, i.e., $s_{2,k-k_\mathrm{d}}, s_{2,k-k_\mathrm{d}-1}, \cdots, s_{2,k-L_1-2}$. The received signal after SIC can be expressed as

$$y_{11,k}^{\mathrm{SIC}} = \mathbf{w}_1^H \mathbf{H}_{11,0} \mathbf{s}_{11,k} + \mathbf{w}_2^H \mathbf{H}_{12,\Delta\tau}^{\mathrm{SIC}} \mathbf{s}_{12,k}^{\mathrm{SIC}} + n_{11,k} \tag{7}$$

where $\mathbf{s}_{12,k}^{\mathrm{SIC}} \triangleq [s_{2,k}\, s_{2,k-1} \cdots s_{2,k-k_\mathrm{d}+1}]^T$ and $\mathbf{H}_{12,\Delta\tau}^{\mathrm{SIC}}$ denote the matrix obtained by removing the $(k_\mathrm{d}+1)$-th and subsequent columns from $\mathbf{H}_{12,\Delta\tau}$. The ANOMA scheme does not require receiver-side equalization, which allows us to directly detect the symbols $\hat{s}_{1,k}$ and $\hat{s}_{2,k}$ from the SIC output in Eq. (7) and the correlator output in Eq. (6).

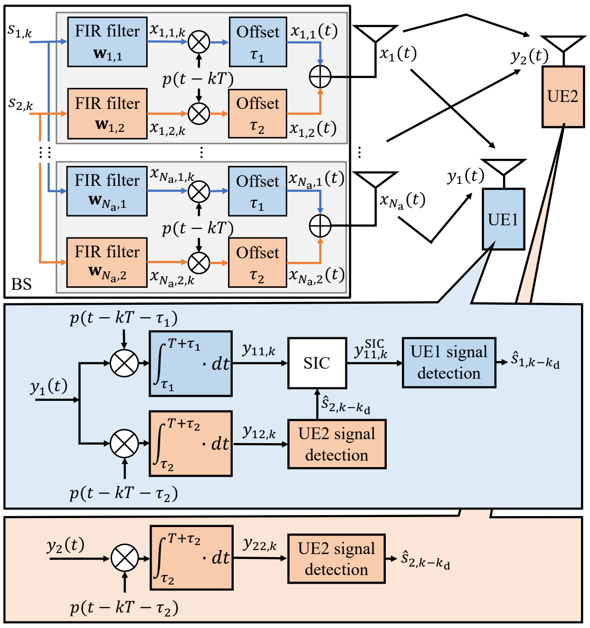


Fig. 1: Block diagram of proposed ANOMA scheme.

III. Transmitter Design

We now describe the transmitter design method, focusing on user fairness. To this end, it is reasonable to maximize the minimum user rate. Since achievable rates are determined jointly by these filter vectors $\mathbf{w}_1$ and $\mathbf{w}_2$, a joint optimization is required. To facilitate this, we define the composite filter vector as $\mathbf{w} \triangleq [\mathbf{w}_1^T\, \mathbf{w}_2^T]^T$. Using this notation, the received signals in Eqs. (7) and (6) can be rewritten as

$$\begin{aligned} y_{11,k}^{\mathrm{SIC}} &= \underbrace{\mathbf{w}^H \left[\mathbf{H}_{11}\right]_{k_\mathrm{d}+2} s_{1,k-k_\mathrm{d}}}_{\text{desired}} \\ &\quad + \underbrace{\mathbf{w}^H \left(\mathbf{H}_{11}\right)_{k_\mathrm{d}+2} \left(\tilde{\mathbf{s}}_{11,k}\right)_{k_\mathrm{d}+2}}_{\text{ISI+IUI}} + n_{11,k}, \end{aligned} \tag{8}$$

$$\begin{aligned} y_{i2,k} &= \underbrace{\mathbf{w}^H \left[\mathbf{H}_{i2}\right]_{L_i+k_\mathrm{d}+2} s_{2,k-k_\mathrm{d}}}_{\text{desired}} \\ &\quad + \underbrace{\mathbf{w}^H \left(\mathbf{H}_{i2}\right)_{L_i+k_\mathrm{d}+2} \left(\tilde{\mathbf{s}}_{i2,k}\right)_{L_i+k_\mathrm{d}+2}}_{\text{ISI+IUI}} + n_{i2,k} \end{aligned} \tag{9}$$

where $\tilde{\mathbf{s}}_{11,k} \triangleq [\mathbf{s}_{11,k}^T \, \mathbf{s}_{12,k}^{\mathrm{SIC}\,T}]^T$, $\tilde{\mathbf{s}}_{i2,k} \triangleq [\mathbf{s}_{i1,k}^T \, \mathbf{s}_{i2,k-1}^T]^T$, $\mathbf{H}_{11} \triangleq \mathrm{diag}(\mathbf{H}_{11,0} \, \mathbf{H}_{12,\Delta\tau}^{\mathrm{SIC}})$, $\mathbf{H}_{i2} \triangleq \mathrm{diag}(\mathbf{H}_{i1,\Delta\tau} \, \mathbf{H}_{i2,0})$. In ANOMA without SIC, $\tilde{\mathbf{s}}_{11,k} = \tilde{\mathbf{s}}_{12,k}$, $\mathbf{H}_{11} \triangleq \mathrm{diag}(\mathbf{H}_{11,0} \, \mathbf{H}_{12,\Delta\tau})$ from Eq. (5). From Eqs. (8) and (9), the achievable rates for UE1 and UE2 are given by

$$R_1 = \log_2\left(1 + \frac{\mathbf{w}^H \mathbf{Q}_{11}^{\mathrm{des}} \mathbf{w}}{\mathbf{w}\mathbf{Q}_{11}^{\mathrm{int}}\mathbf{w} + \sigma_1^2}\right), \tag{10}$$

$$R_2 = \log_2\left(1 + \min_{i=1,2} \frac{\mathbf{w}^H \mathbf{Q}_{i2}^{\mathrm{des}} \mathbf{w}}{\mathbf{w}^H\mathbf{Q}_{i2}^{\mathrm{int}}\mathbf{w} + \sigma_i^2}\right) \tag{11}$$

where $\mathbf{Q}_{ij}^{\mathrm{des}} = [\mathbf{H}_{ij}]_\chi [\mathbf{H}_{ij}]_\chi^H$, $\mathbf{Q}_{ij}^{\mathrm{int}} = (\mathbf{H}_{ij})_\chi (\mathbf{H}_{ij})_\chi^H$. Note that the rate R_2 for UE2 is determined by the minimum of the two decoding SINRs at UE1 and UE2. This is because UE1 performs SIC and therefore the signal intended for UE2 must be reliably decoded at both UE1 and UE2. As a result, the effective rate for UE2 is limited by the worst decoding condition between the two UEs.

The optimization problem can be formulated as

$$\max_{\mathbf{w}} \min_{i=1,2} R_i, \ \text{s.t.} \ \|\mathbf{w}\|^2 \leq P \tag{12}$$

where P denotes the total transmit power of the BS. To solve the above optimization problem, we introduce a semidefinite matrix $\mathbf{W} \triangleq \mathbf{w}\mathbf{w}^H$ and an auxiliary variable ζ. Using these, Eq. (12) can be rewritten as

$$\max_{\mathbf{W},\zeta} \zeta, \tag{13}$$

$$\text{s.t.} \quad \mathrm{Tr}\left\{\left(\mathbf{Q}_{11}^{\mathrm{des}} - \zeta\mathbf{Q}_{11}^{\mathrm{int}}\right)\mathbf{W}\right\} \geq \sigma_1^2\zeta, \tag{14}$$

$$\mathrm{Tr}\left\{\left(\mathbf{Q}_{i2}^{\mathrm{des}} - \zeta\mathbf{Q}_{i2}^{\mathrm{int}}\right)\mathbf{W}\right\} \geq \sigma_i^2\zeta, \ i = 1,2, \tag{15}$$

$$\mathrm{Tr}\left(\mathbf{W}\right) \leq P, \ \mathbf{W} \succeq 0 \tag{16}$$

where the rank-one constraint on $\mathbf{W}$ is relaxed [7]. To solve the above quasi-convex problem, we consider its feasibility from a given value of ζ:

$$\text{find}\,\mathbf{W}, \ \text{s.t.} \ (14), (15), (16). \tag{17}$$

This feasibility problem is semidefinite and can be solved using convex optimization tools such as CVXPY [8]. To obtain the optimal value of ζ, we employ a bisection algorithm as follows:

Step1: Initialize the interval for ζ as $[\zeta_{\mathrm{L}}, \zeta_{\mathrm{U}}] = [0, \mathrm{SINR}_{22}^{\max}]$.
Step2: Set $\zeta = (\zeta_{\mathrm{U}} + \zeta_{\mathrm{L}})/2$.
Step3: Solve the feasibility problem Eq. (17). If feasible, set $\zeta_{\mathrm{L}} = \zeta$, otherwise set $\zeta_{\mathrm{U}} = \zeta$.
Step4: Repeat Step2 and Step3 until $\zeta_{\mathrm{U}} - \zeta_{\mathrm{L}} \leq \epsilon$.

The upper bound $\mathrm{SINR}_{22}^{\max}$ corresponds to the maximum SINR of UE2 when all transmit power P is allocated solely to UE2 and is given by $\mathrm{SINR}_{22}^{\max} = \mathcal{L}_{\max}\left\{\mathbf{Q}_2^{\mathrm{des}}, \mathbf{Q}_2^{\mathrm{ISI}} + (\sigma_2^2/P)\mathbf{I}_{N_{\mathrm{a}}(L_{\mathrm{w}}+1)}\right\}$ where $\mathcal{L}_{\max}\{\mathbf{A},\mathbf{B}\}$ denotes the largest generalized eigenvalue of the matrix pair $(\mathbf{A},\mathbf{B})$, $\mathbf{Q}_2^{\mathrm{des}} = [\mathbf{H}_2]_{k_{\mathrm{d}}+1}[\mathbf{H}_2]_{k_{\mathrm{d}}+1}^H$, $\mathbf{Q}_2^{\mathrm{ISI}} = (\mathbf{H}_2)_{k_{\mathrm{d}}+1}(\mathbf{H}_2)_{k_{\mathrm{d}}+1}^H$. The resulting matrix $\mathbf{W}$ is not guaranteed to be rank-one in general. To extract a feasible filter vector $\mathbf{w}$, we adopt a randomization technique as follows. First, generate candidates by $\mathbf{w}^{(c)} = \sqrt{P}\mathbf{U}\mathbf{\Lambda}^{1/2}\mathbf{v}_c/\|\mathbf{U}\mathbf{\Lambda}^{1/2}\mathbf{v}_c\|$ for $c = 1 \cdots N_{\mathrm{C}}$, where eigendecomposition $\mathbf{W} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$ and $\mathbf{v}_c \sim \mathcal{CN}(0, \mathbf{I}_{N_{\mathrm{a}}(L_{\mathrm{w}}+1)})$ is a random vector. Finally, among the candidates, we select the one that maximizes the minimum user rate.

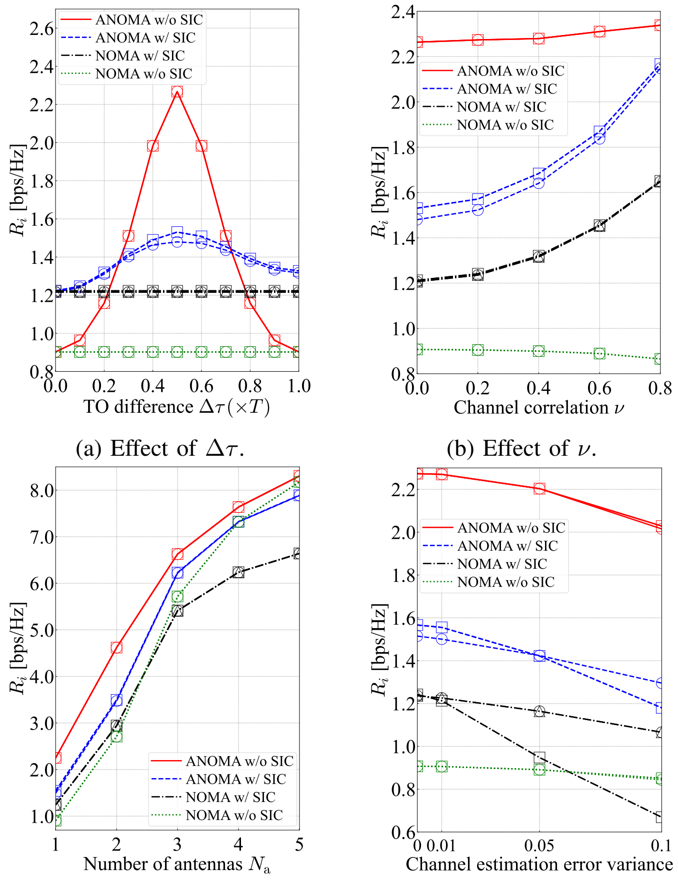
IV. Simulation Results

We show simulation results to evaluate the performance of the ANOMA scheme. Unless otherwise stated, the simulation settings were as follows: $P = 30$ dBm, $N_{\mathrm{a}} = 1$, $L_{\mathrm{w}} = 6$, $L_{\mathrm{h},i} = 3$, $\sigma_i^2 = -103$ dBm, and the pulse $p(t)$ is the time-domain raseid-cosine waveform [9]. The coefficients of the channel impulse response were $h_{n_{\mathrm{a}},i,l} \in \mathcal{CN}(0, \sigma_{\mathrm{h},l}^2)$ and $\sigma_{\mathrm{h},l}^2 = \exp(-\eta l)/\{\sum_{l=0}^{L_{\mathrm{h},i}} \exp(-\eta l)\}, \eta = 0.23$. The path loss model used $128.1 + 37.6\log_{10} r_i$ dB, where r_i km is the distance between BS and UEi [10]. The distance is assumed to be uniformly distributed with $r_1 \in [0.1, 0.2]$ and $r_2 \in [0.3, 0.5]$. The channel correlation coefficient ν [6] is set to 0, i.e., the channels of the users are generated independently.

Figure 2(a) shows the rates as a function of TO difference $\Delta\tau$. For comparison, NOMA ($\Delta\tau = 0$) with and without SIC are shown together. It can be seen that ANOMA achieves higher rates with better user fairness. Furthermore, maximum performance is achieved at $\Delta\tau = T/2$, where IUI power is minimized [6]. This value is adopted in the subsequent simulations. In NOMA, SIC is crucial for mitigating IUI and enhancing fairness. In contrast, for $0.3 \leq \Delta\tau \leq 0.7$, ANOMA without SIC outperforms the case with SIC. In ANOMA, where TO inherently reduces IUI, ISI becomes the primary performance-limiting factor. In such a scenario, the simultaneous reduction of ISI for both UEs by the transmit filter is more effective in terms of fairness than the reduction of IUI by using SIC. In other words, without SIC the transmit filter needs to mitigate ISI for only two received signals, $y_{11,k}^{\mathrm{SIC}}$ and $y_{22,k}$, instead of three ($y_{11,k}^{\mathrm{SIC}}, y_{12,k}\, y_{22,k}$) as in the SIC case. This simplification enables a more effective filter design, resulting in improved user fairness.

Figure 2(b) shows the rates as a function of channel correlation ν. It can be seen that as ν increases, the rate improves when SIC is used. This is because both UEs demodulate the symbols addressed to UE2, and the transmit filter can reduce the ISI of both channels if ν is high. Therefore, when using SIC, pairing UEs with high ν can improve performance. The user rates of ANOMA without SIC also increase slightly as ν increases because the channels of UE1 and UE2 become more similar, enabling more effective ISI suppression by the transmit filter. In contrast, the rate of NOMA without SIC decreases with increasing ν, since when ν is low, the transmit filter can exploit channel diversity to suppress IUI, whereas high ν limits the filter's ability to distinguish between UEs, leading to performance degradation.

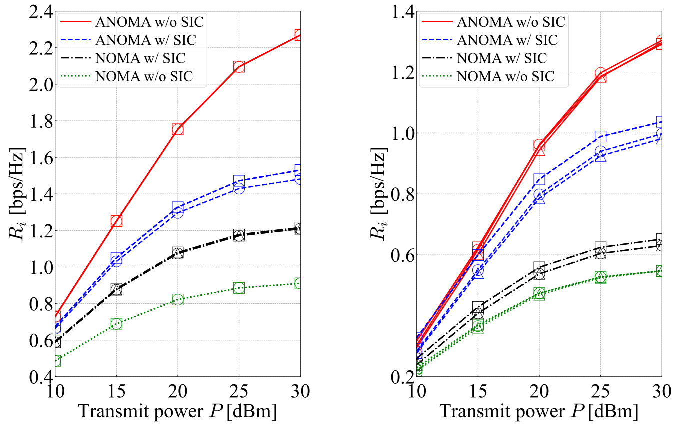
Figure 2(c) shows the rates as a function of the number of antennas N_{a}. It can be observed that as N_{a} increases, the rate achieved by NOMA without SIC approaches that of ANOMA. This improvement in NOMA is due to reduced IUI resulting from spatial separation. Therefore, ANOMA is particularly effective when N_{a} is small.



(a) Effect of $\Delta\tau$. (b) Effect of ν.

(c) Effect of N_a. (d) Effect of imperfect CSI.

Fig. 2: Performance comparison (R_1: □, R_2: ◯).



(a) Two UEs case. (b) Three UEs case.

Fig. 3: Effect of transmit power on rate (R_1: □, R_2: ◯, R_3: △).

Figure 2(d) shows the rates as a function of the channel estimation error variance. As the estimated error increases, the ISI reduction capability of the transmit filter deteriorates, and SIC cannot fully suppress IUI, resulting in a rate reduction. Focusing on the rate of UE1 using SIC, ANOMA can mitigate IUI through asynchronous transmission, resulting in a smaller rate reduction compared to NOMA. Therefore, ANOMA demonstrates greater robustness than NOMA.

Figures 3(a) and 3(b) show the user rates as a function of transmit power. In the three UEs case, the TOs are set to $(\tau_1, \tau_2, \tau_3) = (0, T/3, 2T/3)$; however, this set is not optimal to minimize IUI power, and optimizing TO for three UEs remains future work. The signal determination is performed in the order UE3, UE2, UE1, with SIC applied to UE1 and UE2. A comparison between the two UEs and three UEs cases reveals that the performance gap between ANOMA and NOMA widens, as the benefit of IUI reduction through TO becomes more pronounced with an increase in the number of UEs.

V. Conclusion

In this paper, we proposed an ANOMA scheme with transmit equalization to achieve user fairness. A transmitter was designed to maximize the minimum rate, thereby ensuring fairness among UEs. Simulation results showed that the proposed ANOMA scheme achieves user fairness at higher rates compared to the NOMA scheme. Moreover, with sufficient IUI reduction by TO, fairness is achieved at higher user rates without SIC than with SIC. Future work will focus on analyzing the relationship between IUI power and the necessity of SIC in ANOMA with transmit equalization, developing robust transmitter designs even under conditions of incomplete CSI, and optimizing the TO for cases with three UEs or more.

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